IB Question #2. Answers.

#1. (a) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 14 & 6 \\ 1 & 2 & 0 & -5 \end{array}\right]$

$R_1 - R_3 \to R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 14 & 6 \\ 0 & -1 & 2 & 3 \end{array}\right]$

$3R_1 - R_2 \to R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 4 & -8 & -12 \\ 0 & -1 & 2 & 3 \end{array}\right]$

$\frac{R_2}{-4} \to R_2$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & 2 & 3 \end{array}\right]$

$R_3 - R_2 \to R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow$ The Row operations indicates the three planes intersect to form a line (an infinite Number of solutions)

(b) $-y + 2z = 3$ $\quad x + y + 2z = -2$.

(Define $\boxed{z = t}$) $\quad y = 2z - 3 \Rightarrow \boxed{y = 2t - 3}$. $\quad x + (2t - 3) + 2t = -2$

$\boxed{x = -4t + 1}$

$$\boxed{\begin{aligned} x &= -4t + 1 \\ y &= 2t - 3 \\ z &= t \end{aligned}}$$

the parametric Equations.

$-4\left(\frac{3}{2}\right) + 1$

1.5 0 5

\# 2.

(a)
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 1 & 1 & 1 \\ -1 & 4 & a & 4 \end{array}\right]$$

$R_1 + R_3 \to R_3$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 1 & 1 & 1 \\ 0 & 6 & a-1 & 6 \end{array}\right]$$
$4-1$

$2R_1 - R_2 \to R_2$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 3 & -3 & 3 \\ 0 & 6 & a-1 & 6 \end{array}\right] \to \frac{R_2}{3} \to R_2 \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 6 & a-1 & 6 \end{array}\right]$$

$6R_2 \to R_3$
$\to a_3$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -a-5 & 0 \end{array}\right]$$

$5-5$
$-6+1-a$
$-5+1-a$

When $a = -5 \Rightarrow$ NO unique Solution
But ∞ Number of Solutions

When $a \neq -5$, the system will have an unique solution

(b) If $a = -6$. $\Rightarrow$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore$ The system will always have the solution(s) either ∞ Number or one.

$R_3 + R_2 \to R_2$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_3 + R_1 \to R_1$
$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 - 2R_2 \to R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow x = -4 \quad y = 1 \quad z = 0$$

#3. (a)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 3 \\ 1 & 3 & -1 & \lambda \end{array}\right]$$

$R_1 - R_3 \rightarrow R_3$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 3 \\ 0 & -2 & 2 & 1-\lambda \end{array}\right]$$

$2R_1 - R_2 \rightarrow R_2$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & -2 & 2 & 1-\lambda \end{array}\right]$$

$2R_2 - R_3 \rightarrow R_3$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & \lambda-3 \end{array}\right]$$

$-2-1+\lambda$

$\Rightarrow$ The Row operation Results indicate No unique Solution possible for the System.

(b) (i) when $\lambda \neq 3$ $\Rightarrow$ The system will have ∞ set of solution.

$\lambda = 3$

$x + y + z = 1$ $\qquad x + z + 1 + z = 1$ $\qquad z = \frac{-x}{2}$

$-y + z = -1$ $\qquad y = z + 1$

$\Rightarrow z = y - 1 = \frac{-x}{2}$

General Solution in parametric form $\Rightarrow \boxed{-\frac{x}{2} = y - 1 = z}$

#4.

$$\begin{vmatrix} k & 1 & 2 \\ 0 & -1 & 4 \\ 3 & 4 & 2 \end{vmatrix} = k(-2-16) - \underline{(0-12)} + 2(0+3).$$

$$= -18k + 12 + 6 = 0$$

$$\boxed{k=1}$$

The determinant of the coefficient matrix is a singular. to have no unique solution.

If $k=1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 3 & 4 & 2 & 1 \\ 0 & -1 & 4 & 5 \end{array}\right]$$

$6-2$

$12-1$

$3R_1 - R_2 \to R_2$
$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & -1 & 4 & 11 \\ 0 & -1 & 4 & 5 \end{array}\right]$$

$R_2 - R_3 \to R_3$
$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & -1 & 4 & 11 \\ 0 & 0 & 0 & 6 \end{array}\right]$$

The row operation indicates.

$\Rightarrow$ The system has no solution.

#5 . A (1, 2, 1) B (−3, 1, 4)

C (5, −1, 2) D (5, 3, 7)

(a) $\vec{AB} = \begin{pmatrix} -3-1 \\ 1-2 \\ 4-1 \end{pmatrix} = -4i - j + 3k$

$\vec{AC} = \begin{pmatrix} 5-1 \\ -1-2 \\ 2-1 \end{pmatrix} = 4i - 3j + k$.

(b) $\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -4 & -1 & 3 \\ 4 & -3 & 1 \end{vmatrix} = \begin{matrix} i\ (-1+9) \\ -j\ (-4-12) \\ k\ (+12+\cancel{4}) \end{matrix} = 8i + 16j + 16k$.

$8x + 16y + 16z = F$. A (1, 2, 1)

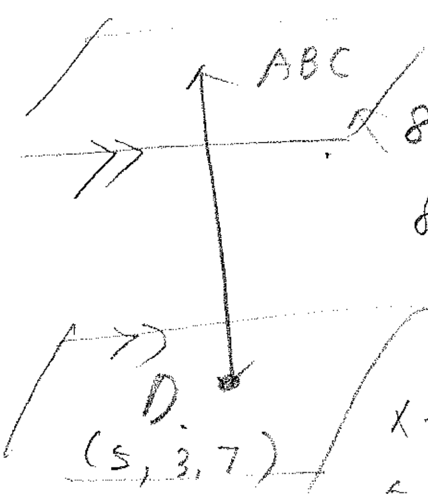
$x + 2y + 2z = \cancel{5}\ 7$

32 $F = \boxed{8 + 32 + 16 = \cancel{40}}$ 56 The plane of ABC : $\boxed{8x + 16y + 16z = \cancel{40}\ 56}$

40
−24
16

Direction D Coordinates

(c) The line ⊥ △ABC ⇒ $\boxed{r = \begin{pmatrix} 8 \\ 16 \\ 16 \end{pmatrix} \lambda + \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix}}$



$8x + 16y + 16z = \cancel{40}\ 56$

$8(x + 2y + 2z = 7)$

1+4+1

$x + 2y + 2z = G \Leftarrow (5, 3, 7)$

$5 + 6 + 14 = 25$

$(x + 2y + 2z = 25)$

$5 + 6 + 14 =$

$\frac{-20}{3}$

Distance: $\left| \frac{7}{3} - \frac{25}{3} \right|$ $\cancel{\left| \frac{2}{\sqrt{1+4+9}} - \frac{(-10)}{\sqrt{1+4+9}} \right|}$

$= \frac{18}{3} = \boxed{6}$

#5. (d)(i) Area of $\Delta ABC = \frac{1}{2}\sqrt{8^2+16^2+16^2} = \frac{1}{2}\sqrt{576}$.

(d) Volume of ABCD

$= \frac{1}{3}$ [Base Area of ABC] $\cdot$ height.

$= \frac{1}{3}\left(\frac{1}{2}\sqrt{576}\right)(6) = \boxed{24 \text{ unit}^3}$

(e) Skip.