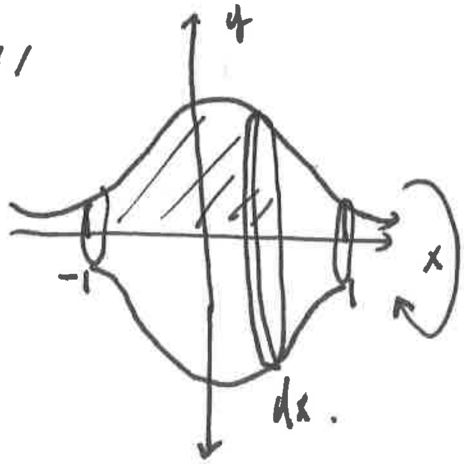
Ch 22. IB Questions (Area and Volumes)

#1



$$V = \pi \int_{-1}^{1} \left(e^{-x^2}\right)^2 dx$$

$$= \pi \int_{-1}^{1} \left(e^{-2x^2}\right) dx \Leftarrow \text{Evaluate it by G.C.}$$

$$\approx \boxed{3.76 \text{ units}^3}$$

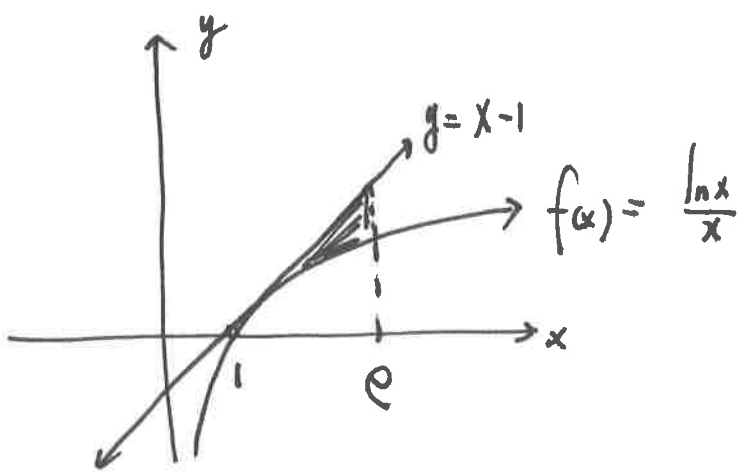
#2. $f(x) = \dfrac{\ln x}{x} \Rightarrow \dfrac{\ln x}{x} = 0 \Rightarrow \ln x = 0 \quad x = 1.$

d) $f'(x) = \dfrac{\frac{1}{x} \cdot x - \ln x}{x^2} \Rightarrow f'(1) = 1$

$\Rightarrow \boxed{y = 1(x-1) = x-1}$

$u = \ln x$
$du = \frac{1}{x} \Rightarrow \int u\,du$

e)



$$A = \int_{1}^{e} \left((x-1) - \frac{\ln x}{x}\right) dx.$$

$$= \left[\frac{x^2}{2} - x - \frac{1}{2}(\ln x)^2\right]_{x=1}^{x=e}$$

$$= \left(\frac{e^2}{2} - e - \cancel{\frac{1}{2}}\right) - \cancel{\left(\frac{1}{2} - 1\right)}$$

$$= \boxed{\frac{e^2}{2} - e}$$

3.

(a) $y = x + 2\cos x = x$

$\Rightarrow 2\cos x = 0$

$\Rightarrow \cos x = 0$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$.

Intersections

$\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ & $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

(b) $$V = \pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[x^2 - (x + 2\cos x)^2 \right] dx$$

$$= \pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\cancel{x^2 - x^2} - 4x\cos x - 4\cos^2 x \right] dx$$

$$= \pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[-4\left(x\cos x + \cos^2 x \right) \right] dx$$

$\Leftarrow$ To evaluate this, Use of calculator is okay.

u	du
x	cos x
1 (+)	sin x
0 (−)	− cos x

$$= \pi \left[-4 \left[x\sin x + \cos x + \frac{1}{2}\left(x + \frac{1}{2}\sin 2x \right) \right] \right]_{x=\frac{\pi}{2}}^{x=\frac{3\pi}{2}}$$

$\cos^2 x = \frac{1}{2}\left[1 + \cos 2x \right]$

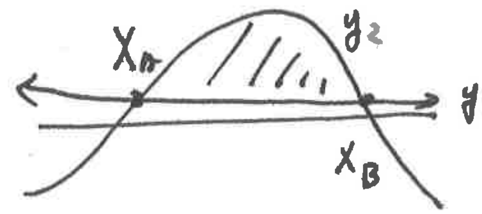
$$= -4\pi\left[-\frac{3\pi}{2} + \frac{1}{2}\left(\frac{3\pi}{2}\right) \right] + 4\pi\left[\frac{\pi}{2} + \frac{1}{2}\left(\frac{\pi}{2}\right) \right]$$

$$= \frac{\overset{6}{\cancel{12}}\pi^2}{\cancel{2}} - 3\pi^2 + \frac{\overset{2}{\cancel{4}}\pi^2}{\cancel{2}} + \pi^2$$

$$= 5\pi^2 + \pi^2 = \boxed{6\pi^2}$$

#4. a) $x^2 e^{-x} = 1 - 2\sin x$. $2 \le x \le 7$

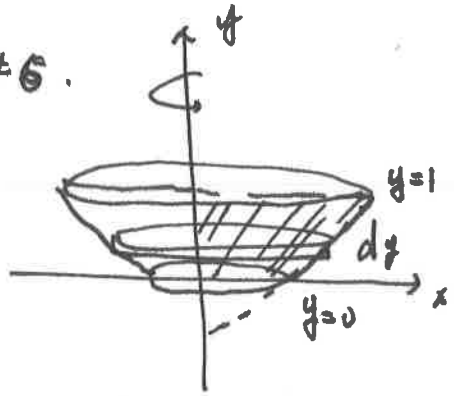
$x_A \approx 2.87$ $x_B \approx 6.78$



b) $A = \int_{2.87}^{6.78} \left[(1 - 2\sin x) - x^2 e^{-x} \right] dx \approx \boxed{6.76 \text{ units}^2}$

↑ Evaluate by G.C.

#6.



$y = \sin(x-1)$

$x = (\arcsin y) + 1$

$\rightarrow V = \pi \int_{y=0}^{y=1} \left[\arcsin y + 1 \right]^2 dy.$

↑ Evaluate by G.C.

$\approx \boxed{8.20 \text{ units}^3}$

#7.

$a(v) = \frac{1}{40}(60 - v) \ \frac{m}{s^2}$

$a = \frac{dv}{dt}$

$\Rightarrow \frac{dv}{dt} = \frac{1}{40}(60 - v)$

$\Rightarrow \int \frac{dv}{60 - v} = \int \frac{1}{40} dt$

$-\ln(60 - v) = \frac{1}{40}t + C$

Rest

$t = 0 \quad v = 0$

$\Rightarrow C = -\ln 60.$

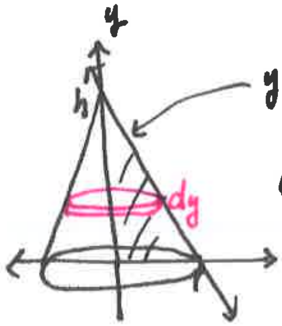
$$-\ln(60-x) = \frac{1}{40}t - \ln 60 \quad \Leftarrow t = 30.$$

$$-\ln(60-x) = \frac{3}{4} - \ln 60$$

$$\ln(60-x) = \ln 60 - \frac{3}{4}$$

$$60 - x = e^{\left(\ln 60 - \frac{3}{4}\right)} \Rightarrow x = \boxed{60 - e^{(\ln 60 - 3/4)}} \ \frac{m}{s}.$$

#8.



$y = h - \frac{h}{r}x \Rightarrow \frac{h}{r}x = h - y$

disk $\quad x = \frac{r}{h}(h-y)$

$$V = \pi\int_0^h \left(\frac{r}{h}(h-y)\right)^2 dy \Rightarrow \frac{r^2}{h^2}\pi\int_0^h (h^2 - 2hy + y^2)\,dy$$

$$= \frac{r^2}{h^2}\pi\left[h^2y - hy^2 + \frac{1}{3}y^3\right]_{y=0}^{y=h}$$

$$= \frac{r^2}{h^2}\pi\left[\cancel{h^3} - \cancel{h^3} + \frac{1}{3}h^3\right] = \boxed{\frac{1}{3}r^2h\pi} \Leftarrow$$ Volume for a cone. Formula.

#10. (a) $A = \int_{\frac{1}{6}}^{1}\left(\frac{k}{x} - \frac{1}{x}\right)dx = \left[k\ln x - \ln x\right]_{\frac{1}{6}}^{1}$

$= 0 - \left[k\ln\frac{1}{6} - \ln\frac{1}{6}\right] = -\left[-k\ln 6 + \ln 6\right] = \ln 6\,[k-1]$

(b) $B = \int_1^{\sqrt{6}}\left(\frac{k}{x} - \frac{1}{x}\right)dx = \left[k\ln x - \ln x\right]_1^{\sqrt{6}} = k\ln\sqrt{6} - \ln\sqrt{6} = \frac{1}{2}\ln 6\,[k-1]$

(c) $A : B \Rightarrow \ln 6\,[k-1] : \frac{1}{2}\ln 6\,[k-1] \Rightarrow \boxed{1 : \frac{1}{2}}$