#1. $v = \cos(t^2)$

(a) $t = 0 \quad S = 0$

(i) Displacement $= \int_0^3 \cos(t^2)\,dt \approx \boxed{.703 \text{ m}}$

(ii) Total Distance $= \int_0^3 |\cos(t^2)|\,dt \approx \boxed{2.05 \text{ m}}$

(b) $1 = \int_0^{t_2} |\cos(t^2)|\,dt$ $\boxed{t_2 \approx 1.39 \text{ sec}}$

$\text{nSolve}\left(\int_0^{t_2} |\cos(t^2)|\,dt = 1,\ t_2 \right) \Rightarrow$

2. $v = 9t - 3t^2 \quad 0 \le t \le 5$

$\boxed{t = 0 \quad S = 3\,m}$.

(a) $\text{Displacement} = \int_0^4 (9t - 3t^2)\,dt$.

$$= \frac{9}{2}t^2 - t^3 \Big|_{t=0}^{t=4} = \boxed{8\,m}$$

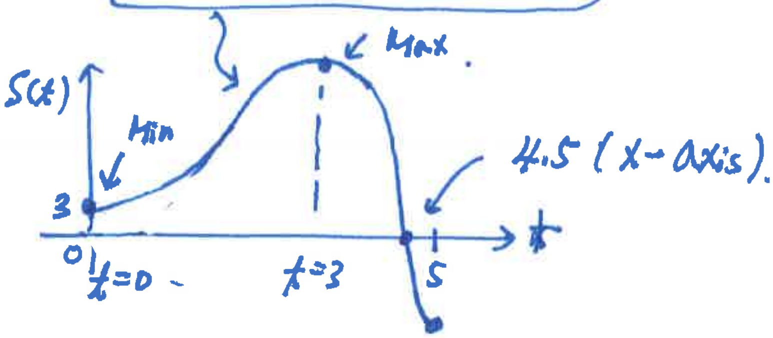
$0 \le t \le 5$.

(b) $\boxed{S(t) = \frac{9}{2}t^2 - t^3 + 3}$

$v = 9t - 3t^2$

$= 3t(3 - t)$

$t = 0 \quad t = 3$



$S = a + b\cos\left(\frac{2\pi t}{5}\right) \qquad t > 5$

(c) $t = 7.5 \qquad S = 16.5$

$\Rightarrow \boxed{16.5 = a + b\cos\left(\frac{2\pi(7.5)}{5}\right)} \cdots ① \Rightarrow 16.5 = a - b$

$t = 5 \qquad S = \frac{9}{2}(5)^2 - (5)^3 + 3 = -9.5$

$\boxed{-9.5 = a + b\cos\left(\frac{2\pi(5)}{5}\right) \cdots} ② \Rightarrow -9.5 = a + b$

Solve the system of Equations $\left.\begin{array}{l} 16.5 = a - b \\ -9.5 = a + b \end{array}\right) \Rightarrow \begin{array}{l} a = 3.5 \\ b = -13 \end{array}$

$3 = \frac{9}{2}t^2 - t^3 + 3$

(d) $3 = 3.5 - 13\cos\left(\frac{2\pi t}{5}\right)$. $\qquad t_1 = \frac{9}{2} \qquad t_2 = 6.22$

#3

$$v = \frac{-1}{s^2} = -s^{-2} \quad s > 0$$

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot v.$$

$$\frac{dv}{ds} = 2s^{-3} = \frac{2}{s^3}$$

$$a = \left(\frac{2}{s^3}\right)\left(\frac{-1}{s^2}\right) = \boxed{\frac{-2}{(s)^5}}$$

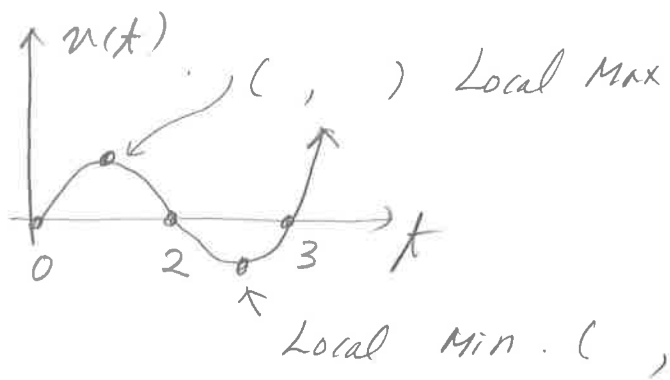
$$a(50) = \boxed{\frac{-2}{(.5)^5}} = -64 \frac{m}{sec}$$

#5.

$$v_A = t^3 - 5t^2 + 6t \qquad a(t) = 3t^2 - 10t + 6 = 0.$$

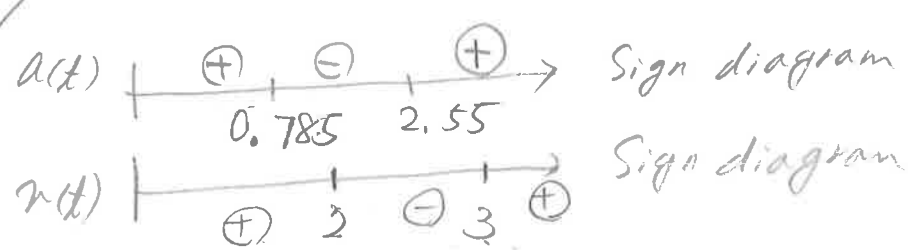
$$= t(t^2 - 5t + 6) = t(t-2)(t-3)$$

a)



b) Velocity is inceasing. $(0, \ ) \cup (\ , \infty)$

c) Magnitude of the velocity (speed)



$$(0, 0.785) \cup (2, 2.55) \cup (3, \infty)$$

#5 Continue

d) $x_A = \int (t^3 - 5t^2 + 6t)\,dt$

$= \frac{1}{4}t^4 - \frac{5}{3}t^3 + \frac{6}{2}t^2 + C \Leftarrow t=0 \quad S=0$

$C = 0$

$\boxed{x_A = \frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2}$

Challenge

e) $a_B = -2v_B$ when $t=0$, $v_B = -20$ and $x_B = 20$

$a_B = \frac{dv_B}{dt}$

$\Rightarrow \frac{dv_B}{dt} = -2v_B$

$\int \frac{dv_B}{v_B} = \int -2\,dt$

$\ln v_B = -2t + C_1$

$v_B = e^{-2t+C} = e^{-2t} \cdot e^{C} = A \cdot e^{-2t}$

$-20 = A \cdot e^0 \Rightarrow A = -20$

$\boxed{v_B = -20 \cdot e^{-2t}}$

f) $x_B = \int -20 \cdot e^{-2t}\,dt = 10e^{-2t} + C_2$

$20 = 10 \cdot e^0 + c \qquad \boxed{C_2 = 10}$

$\boxed{x_B = 10e^{-2t} + 10}$

$\Rightarrow 10e^{-2t} + 10 = \frac{1}{4}t^4 - \frac{5}{4}t^3 + 3t^2$ Solve for t by G.C.

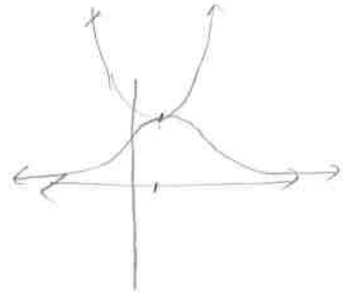
#4.

(a) $4x^2-4x+5$

$= 4(x^2-x \quad) + 5$

$= 4(x^2-x+\frac{1}{4}) + 5 - (4)(\frac{1}{4})$

$= \boxed{4(x-\frac{1}{2})^2+4}$

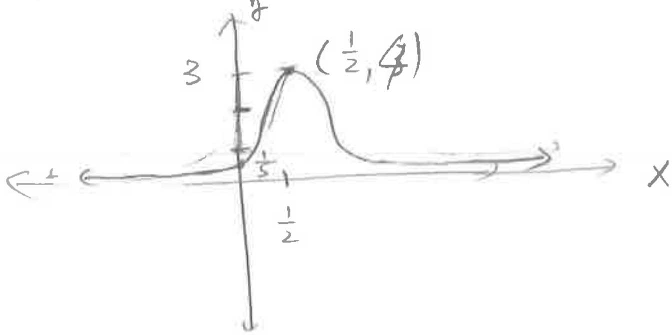


(b) Vertical Dilation by a factor of 4, H.T: Right $\frac{1}{2}$, V.T: up 3.

(c) $f(x) = \dfrac{1}{4x^2-4x+5} = \dfrac{1}{4(x-\frac{1}{2})^2+}$

&

(d)



Range: $0 < f(x) \le 4 \Rightarrow (0, 4]$

(e) $\int f(x)dx = \int \dfrac{1}{4(x-\frac{1}{2})^2+4}dx$

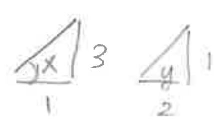
$u = x-\frac{1}{2}$ $\qquad = \frac{1}{4}\int \dfrac{1}{(x-\frac{1}{2})^2+1}dx = \frac{1}{4}\int \dfrac{1}{u^2+1}du$

$du = dx$

(f) $\int_1^{3.5} \dfrac{1}{4x^2-4x+5}dx = \frac{1}{4}\int_{\frac{1}{2}}^{3} \dfrac{1}{u^2+1}du = \frac{1}{4}\arctan u\Big|_{\frac{1}{2}}^{3}$

When $x=1$ $u=\frac{1}{2}$ $\qquad = \frac{1}{4}[\arctan 3 - \arctan\frac{1}{2}]$

$x=3.5$ $u=3$



$\Rightarrow \frac{1}{4}[x - y] = \frac{\pi}{16}$ $\qquad \dfrac{\tan x - \tan y}{1-\tan x \cdot \tan y}$

$\Rightarrow \tan[x - y] = \tan\frac{\pi}{4}$ $\qquad = \dfrac{3-\frac{1}{2}}{1+3\cdot\frac{1}{2}} = \dfrac{2.5}{2.5} = 1$

#6. (a) $v(t) = -10t$

(i) $a(t) = -10 \ \frac{m}{s^2}$

(ii) $v(10) = (-10)(10) = -100 \ \frac{m}{s}$

(iii) $S(100)$

$\Rightarrow S(t) = \int v(t) = -5t^2 + C_1 \Leftarrow S(0) = 1000$

$C_1 = 1000$

$\Rightarrow \boxed{S(t) = -5t^2 + 1000}$

(b) $t = 10 \qquad a(t) = -10 - 5v \qquad t \geq 10$

$\frac{dt}{dv} = \frac{1}{\frac{dv}{dt}} = \frac{1}{a(t)} = \boxed{\frac{1}{-10-5v}}$.

(c) $\int dt = \int \left(\frac{1}{-10-5v}\right) \cdot dv$

$t = \frac{-1}{5} \ln |-10-5v| + C_2 \qquad \left(\begin{array}{l} t = 10 \\ v = -100 \end{array} \right)$

$10 = \frac{-1}{5} \ln |-10 - 5(-100)| + C_2$

$10 + \frac{1}{5} \ln |490| = C_2$

$\Rightarrow \boxed{t = \frac{-1}{5} \ln |-10-5v| + 10 + \frac{1}{5} \ln (490)}$

(d) $S(t) = \int (v)\,dt$

$$t = \frac{1}{5}\ln(490) - \frac{1}{5}\ln(-10-5v) + 10.$$

$$t = \frac{1}{5}\ln\left[\frac{490}{-10-5v}\right] + 10 \Rightarrow \text{Solve for } v.$$

$$5(t-10) = \ln\left(\frac{490}{-10-5v}\right) \Rightarrow \frac{490}{-10-5v} = e^{5(t-10)}$$

$$\Rightarrow \frac{10+5v}{-490} = e^{5(10-t)}$$

$$\Rightarrow \frac{10+5v}{5} = -490\, e^{5(10-t)}$$

$$\Rightarrow 2 + v = -98\, e^{5(10-t)}$$

$$\boxed{v = -98\, e^{5(10-t)} - 2}$$

(e) $S(t) = \int \left(-98\, e^{5(10-t)} - 2\right) dt$

$$S(t) = \frac{98}{5} e^{5(10-t)} - 2t + C_3 \Leftarrow \begin{pmatrix} t = 10 \\ S = 500 \end{pmatrix}$$

$$C_3 = 500.4$$

$$\boxed{S(t) = 19.6\, e^{5(10-t)} - 2t + 500.4}$$

(d) $0 = 19.6\, e^{5(10-t)} - 2t + 500.4$ $\boxed{t = 250}$

Kinematics IB Questions Solutions.

#1.

(a) (i) Displacement $= \int_0^3 \cos(t^2)\,dt \approx .703$ m

(ii) Total Distance $= \int_0^3 |\cos(t^2)|\,dt \approx 2.05$ m.

(b) $1 = \int_0^t \cos(t^2)\,dt \Leftarrow$ Use G.C.

$$\boxed{t \approx 1.39 \text{ Sec.}}$$

Challenge

$$a = \frac{2s}{s^2+1}$$

(b) $v = \sqrt{2\ln(s^2+1)+4-2\ln 2}$

$\approx$.

(a) $a = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = v\cdot\frac{dv}{ds} = \frac{2s}{s^2+1}$

$$\int v\cdot dv = \int \frac{2s}{s^2+1}\,ds$$

$$\boxed{\frac{1}{2}v^2 = \ln(s^2+1)+c}$$

$u = s^2+1$

$du = 2s\cdot du$

$v = \sqrt{2\ln(s^2+1)+c}$

$\uparrow$ $v=2$ when $s=1$

$2 = \sqrt{2\ln(2)+c}$

$4 = 2\ln 2 + c$

$c = 4 - 2\ln 2$

$$\Rightarrow \boxed{v = \sqrt{2\ln(s^2+1)+4-2\ln 2}}$$