

IB Questions (Rational Functions)

Key

①

#1. $f(x) = \frac{x+a}{bx+c}$

V.A: $x = -4$ $bx+c = -0$

$bx = -c$

$x = \frac{-c}{b} = -4$

$\frac{c}{b} = 4$ $c = (4)(b) = (4)(-\frac{1}{2}) = -2$

H.A: $y = -2$ $\Rightarrow \frac{1}{b} = -2 \Rightarrow b = -\frac{1}{2}$

$(\frac{2}{3}, 1) \Rightarrow f(\frac{2}{3}) = \frac{\frac{2}{3} + a}{(\frac{2}{3})(-\frac{1}{2}) - 2} = 1 \Rightarrow \frac{\frac{2}{3} + a}{-\frac{1}{3} - \frac{6}{3}} = 1 \Rightarrow \frac{2}{3} + a = \frac{-7}{3}$

#2. $\frac{2x}{|x-1|} < 1 \Rightarrow 2x < |x-1|$

$x \neq 1$

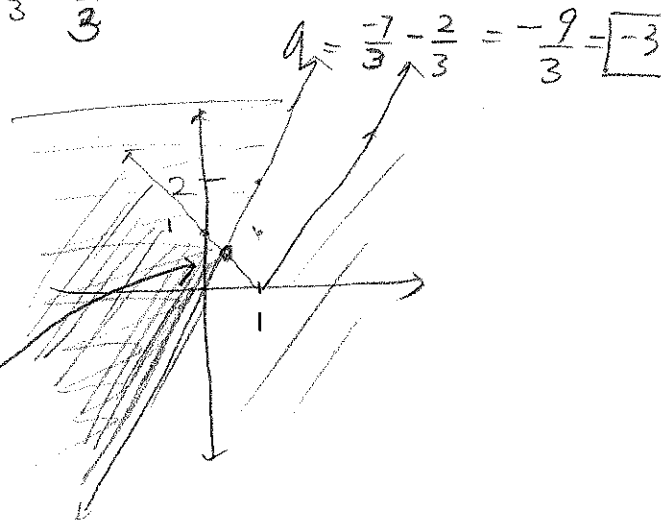
$2x < y_1$

$y_2 < |x-1|$

$2x = -x + 1$

$3x = 1 \quad x = \frac{1}{3}$

$\therefore (-\infty, \frac{1}{3})$



OR

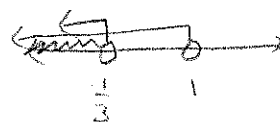
When $x < 1 \Rightarrow$

$\frac{2x}{2x} < \frac{-(x-1)}{2x}$

$2x < -x + 1 \Rightarrow$

$3x < 1$

$x < \frac{1}{3}$

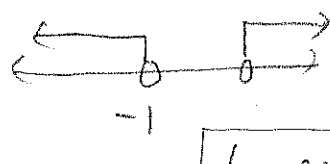


$x < \frac{1}{3}$

When $x > 1 \Rightarrow$

$2x < x - 1 \Rightarrow$

$x < -1$



no solution

#3. $y = \frac{3x-2}{2x-1} \quad x \in \mathbb{R} \quad x \neq \frac{1}{2}$

a) $x = \frac{3y-2}{2y-1}$

$$x(2y-1) = 3y-2$$

$$2xy - x = 3y - 2$$

$$2xy - 3y = x - 2$$

$$y(2x-3) = x-2$$

$$f^{-1}(x) \Rightarrow y = \frac{x-2}{2x-3}$$

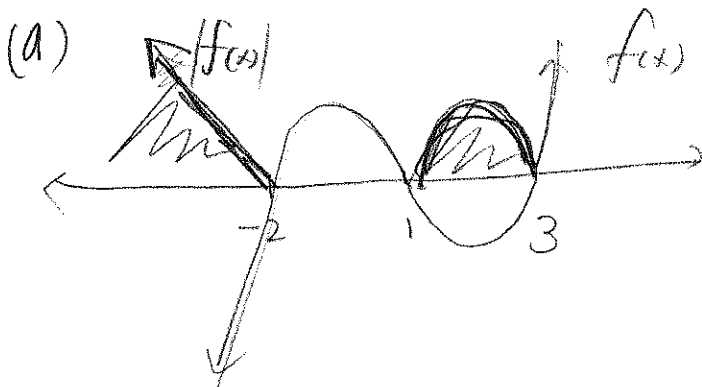
b) $y = \frac{3x-2}{2x-1} = \frac{3}{2} + \frac{-\frac{1}{2}}{2x-1}$

$$\begin{array}{r} \frac{3}{2} \\ 2x-1 \overline{) 3x-2} \\ \underline{3x-1} \\ -1 \\ \underline{-1} \\ 0 \end{array} \quad \begin{array}{l} -2 + \frac{3}{2} \\ -\frac{4}{2} + \frac{3}{2} \\ -\frac{1}{2} \end{array}$$

$$a = \frac{3}{2} \quad b = -\frac{1}{2}$$

#4. $f(x) = (x+1)(x-1)(x-3)$

$$|f(x)| = |(x+1)(x-1)(x-3)|$$



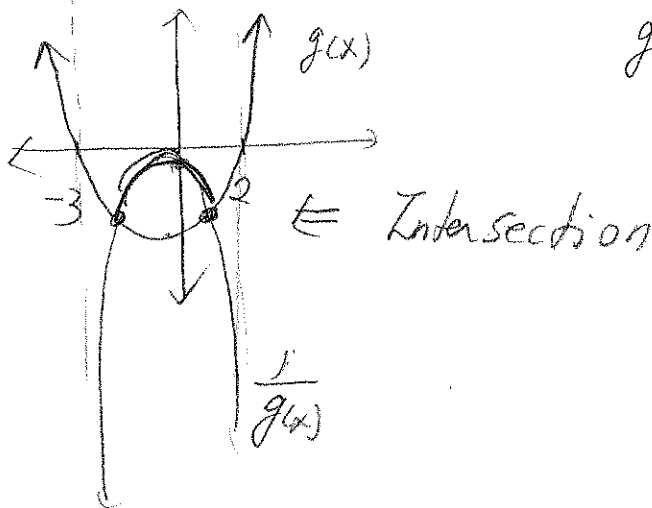
$$\Leftrightarrow \begin{array}{l} f(x) < |f(x)| \\ f(x) < |f(x)| \end{array}$$

$$(-\infty, -2) \cup (1, 3)$$

(b) $g(x) = x^2 + x - 6 \quad x \in \mathbb{R}$

$$\frac{1}{g(x)} = \frac{1}{(x+3)(x-2)}$$

#7 (D)



$$g(x) < \frac{1}{g(x)}$$

#5. $f(x) = 2x + 3$ (a) (i) $(g \circ f)(x) = \frac{1}{2x+3}$

$$g(x) = \frac{1}{x}$$

Domain: $x \in \mathbb{R} \quad x \neq -\frac{3}{2}$

$$g^{-1} \Rightarrow \begin{aligned} x &= \frac{1}{y} \\ y &= \frac{1}{x} \end{aligned}$$

(ii) $(f \circ g)(x) = \frac{2}{x} + 3 = \frac{3x+2}{x}$

Domain: $x \in \mathbb{R} \quad x \neq 0$

(b) $y = (g^{-1} \circ f \circ g)(x)$

$$y = \frac{1}{\frac{3x+2}{x}} = \frac{x}{3x+2}$$

$$f(x) = y \Rightarrow 2x+3 = \frac{x}{3x+2} \quad x \neq -\frac{2}{3}$$

$$\Rightarrow (2x+3)(3x+2) = x$$

$$6x^2 + 13x + 6 = x$$

$$6x^2 + 12x + 6 = 0$$

$$6(x^2 + 2x + 1) = 0$$

$$6(x+1)^2 = 0 \quad (x = -1) \quad (y = 1) \Rightarrow (-1, 1)$$

#6 H.A: $y = 2$

V.A: $x = -2$

y-int: $(0, -2)$

x-int: $(2, 0)$

$$y = \frac{2(x-2)}{(x+2)}$$

$$\left(y = \frac{a+x}{b+cx} \right) = \frac{(2x-4) \div 2}{(x+2) \div 2}$$

$$= \frac{-2+x}{1+\frac{1}{2}x} \quad \begin{cases} a = -2 \\ b = 1 \\ c = \frac{1}{2} \end{cases}$$

$$\Rightarrow (-1, 1)$$