IB Questions.

#1. $8y\ln x - 2x^2 + 4y^2 = 7$

$$8\cdot\frac{dy}{dx}\cdot\ln x + 8y\cdot\frac{1}{x} - 4x + 8y\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[8\ln x + 8y\right] = 4x - 8y\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = \frac{4x - 8y\frac{1}{x}}{8\ln x + 8y}.$$

$x=1 \Rightarrow \cancel{8\cdot y(\ln 1)}^{0} - 2(1)^2 + 4y^2 = 7$

$4y^2 = 7+2 = 9$

$y^2 = \frac{5}{4} \Rightarrow y = \pm\frac{3}{2}$

1) $(1, \frac{3}{2})$ $\quad \left.\frac{dy}{dx}\right|_{(1,\frac{3}{2})} = \boxed{\frac{-2}{3}}$

2) $(1, -\frac{3}{2})$ $\quad \left.\frac{dy}{dx}\right|_{(1,-\frac{3}{2})} = \boxed{\frac{-4}{3}}$

#2. $f(x) = (\ln(x-2))^2$

$$\frac{df}{dx} = 2\ln(x-2)\cdot\frac{1}{x-2}. = \boxed{\frac{2\ln(x-2)}{x-2}}$$

#3. $f(x) = \frac{\ln x}{x}$ $\qquad x \geq 1$

$$\frac{df}{dx} = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2}, \quad x \geq 1$$

$$\frac{df}{dx} = 0 = \frac{1-\ln x}{x^2} \Rightarrow 1 - \ln x = 0 \Rightarrow \boxed{x = e}$$

#4.

$$f(x) = \frac{x^2}{e^x}$$

$$\frac{df}{dx} = \frac{2x \cdot \cancel{e^x} - x^2 \cdot \cancel{e^x}}{(e^x)^{\cancel{2}}}$$

$$= \frac{2x - x^2}{e^x}$$

$$\frac{df}{dx} = 0 = \frac{2x - x^2}{e^x} = \frac{x(2-x)}{e^x}$$

$x = 0, \quad x = 2$ where the gradient of tangent is '0'.

$$\boxed{\Rightarrow (0,0) \quad \Rightarrow \left(2, \frac{4}{e^2}\right)}$$

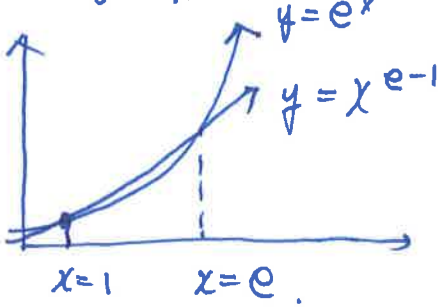
#5. $f(x) = e^x - x^e$

(i) $\dfrac{df}{dx} = e^x - e \cdot x^{e-1}$

(ii) $e^x - e \cdot x^{e-1} = 0$

$\boxed{x = 1, \quad x = e}$ ⇐ Solve by G.C.

Use of GC.



#7. 1) $\displaystyle\lim_{x \to 2^-} (2x-1) = \lim_{x \to 2^+} ax^2 + bx - 5$

$3 = 4a + 2b - 5 \quad \Rightarrow \boxed{2a + b = 4}$

2) $\displaystyle\lim_{x \to 2^-} (2x-1)' = \lim_{x \to 2^+} (ax^2 + bx - 5)'$

$2 = 2a(2) + b. \quad \Rightarrow \boxed{4a + b = 2}$

Solve system of Equations

$$\boxed{\begin{aligned} a &= -1 \\ b &= 6 \end{aligned}}$$

#7

$4-1=3$

(c) $f(x) = \begin{cases} 2x-1 & x \le 2 \\ -x^2+6x-5 & 2 < x < 3 \end{cases}$

	D	R
$2x-1$	$x \le 2$	$y \le 3$
$-x^2+6x-5$	$2< x <3$	$3 < y < 4$

$\Rightarrow -x^2+6x-5 = y$

$-y^2+6y-5 = x$

$y^2-6x+5 = -x$ $\quad +4 \quad +4$

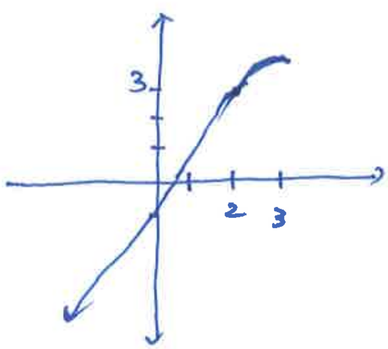
$(y-3)^2 = (4-x)$

$y = \pm\sqrt{4-x}$

$y = \cancel{\pm}\sqrt{4-x} + 3$

$$\boxed{f^{-1}(x) \begin{cases} \frac{x+1}{2}, & x \le 3 \\ \sqrt{4-x}+3 & 3 < x < 4 \end{cases}}$$

b)



one to one since pass V. line and H. line tests.

a) $(x+h)^3 = x^3+3x^2h+3h^2x+h^3$

#8. b) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(\cancel{x^3}+3x^2h+3h^2x+h^3)-(\cancel{x^3})}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3hx+h^2)}{\cancel{h}} = \boxed{3x^2}$

#9. $f(x) = \left(\frac{x}{1-x}\right)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}\left(\frac{x}{1-x}\right)^{-\frac{1}{2}}\left(\frac{(1-x)-(x)(-1)}{(1-x)^2}\right)$

$= \frac{1}{2}\left(\sqrt{\frac{1-x}{x}}\right)\frac{1}{(1-x)^2} = \frac{\sqrt{1-x}}{2\sqrt{x}\,(1-x)^2}$

$= \frac{\sqrt{1-x}\cdot\sqrt{x}}{2x(1-x)^2}$

$\frac{df}{dx} = 0$

$\sqrt{1-x}\cdot\sqrt{x} = 0$

$x \ne 0, \quad x \ne 1$

No solution.