Implicit Differentiation Notes (Day two)

Warm up: Find $\frac{dy}{dx}$

a. $\frac{d}{dx}(7x^2+3\sqrt{y}) = \frac{d}{dx}(2x)$

$14x + (3)(\frac{1}{2})y^{-\frac{1}{2}} \cdot \boxed{\frac{dy}{dx}} = 2$

$-14x \qquad\qquad -14x$

$\frac{2\sqrt{y}}{3} \cdot \frac{3}{2\sqrt{y}} \cdot \boxed{\frac{dy}{dx}} = (2-14x) \cdot \frac{2\sqrt{y}}{3} = \boxed{\frac{2\sqrt{y}(2-14x)}{3}}$ ✓

b. $\frac{d}{dx} 5x^3y^2 = \frac{d}{dx} 20$

$(5x^3)'y^2 + (5x^3)(y^2)' = 0$

$15x^2 \cdot y^2 + (5x^3)(2y)\boxed{\frac{dy}{dx}} = 0$.

$\Rightarrow \frac{dy}{dx} = \frac{-15x^2y^2}{10x^3y} = \boxed{\frac{-3y}{2x}}$

Example 1) Given $x^2+y^2=26$, find the slope of the tangent line to the curve at x=1.

slope(s)

1) Find $\frac{dy}{dx}$.

$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(26)$

$2x + 2y \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} = \boxed{-\frac{x}{y}}$

2) Find the values of y when $x=1$.

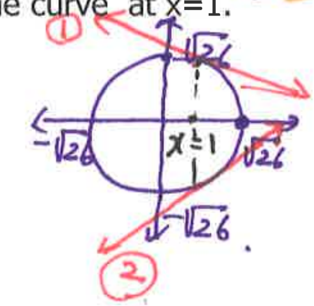
$(1)^2 + y^2 = 26$

$y^2 = 25 \Rightarrow y = \pm 5$.

3) $(1,5)$ and $(1,-5)$

$\frac{dy}{dx}\Big|_{(1,5)} = \boxed{\frac{-1}{5}} \qquad \frac{dy}{dx}\Big|_{(1,-5)} = \boxed{\frac{1}{5}}$

Example 2) Given $x^2+y^2=26$, find the equation(s) of the tangent lines to the curve at x=1.



line: $y = mx + b$

$\boxed{(y-y_1) = m(x-x_1)}$ Tangent line

$\Rightarrow$ 1) $m = \frac{-1}{5}$ $(1,5) \Rightarrow \boxed{y-5 = \frac{-1}{5}(x-1)}$

OR $\boxed{y = -\frac{1}{5}x + \frac{26}{5}}$

2) $m = \frac{1}{5}$ $(1,-5) \Rightarrow \boxed{y+5 = \frac{1}{5}(x-1)}$ OR $\boxed{y = \frac{1}{5}x - \frac{26}{5}}$

Example 3) If $4x^2 - 3y^2 = 37$, find the equation(s) of the tangent lines to the curve at x=4.

1) Find $\frac{dy}{dx}$

$8x - 6y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{4x}{3y}$

2) Find the y values when $x=4$

$(4)(4)^2 - 3y^2 = 37$

$y = \pm 3$

3) $\frac{dy}{dx}\Big|_{(4,3)} = \frac{(4)(4)}{(3)(3)} = \frac{16}{9}$

$\Rightarrow$ Equation: $\boxed{y-3 = \frac{16}{9}(x-4)}$

$\frac{dy}{dx}\Big|_{(4,-3)} = \frac{16}{-9}$

$\Rightarrow$ Equation: $\boxed{y+3 = \frac{-16}{9}(x-4)}$