Integrals with Trig Identities - Solutions

4. $\int \sin^2 x \, dx$

$\int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$\boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + C}$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

5. $\int \sin^3 x \, dx$

$\int (1 - \cos^2 x)\sin x \, dx$

$\int \sin x \, dx - \int \cos^2 x \sin x \, dx$

$\boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$

6. $\int \cos^2 5x \, dx$

$\int \left(\frac{1}{2} + \frac{1}{2}\cos 10x\right) dx$

$\boxed{\frac{1}{2}x + \frac{1}{20}\sin 10x + C}$

7. $\int \sec^3 x \tan x \, dx$

$\int \sec^2 x (\sec x \tan x) \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int u^2 \, du$

$\frac{1}{3}u^3 + C$

$\boxed{\frac{1}{3}\sec^3 x + C}$

8. $\int \frac{\sec^2 7x}{\tan 7x} \, dx$

$u = \tan 7x$

$du = 7\sec^2 7x \, dx$

$\frac{1}{7} du = \sec^2 7x \, dx$

$\int \frac{1}{7} \cdot \frac{1}{u} \, du$

$\frac{1}{7}\ln|u| + C = \boxed{\frac{1}{7}\ln|\tan 7x| + C}$

9. $\int \tan x \, dx$

$\int \frac{\sin x}{\cos x} \, dx$ $\quad u = \cos x$, $du = -\sin x \, dx$

$\int -\frac{1}{u} \, du$

$-\ln|u| + C$

$\boxed{-\ln|\cos x| + C}$

10. $\int \frac{\sin x}{(1+\cos x)^2} \, dx$ $\quad u = 1 + \cos x$, $du = -\sin x \, dx$

$\int -u^{-2} \, du$

$+ u^{-1} + C$

$\boxed{\frac{1}{1+\cos x} + C}$

11. $\int \sin x \, e^{\cos x} \, dx$ $\quad u = \cos x$, $du = -\sin x \, dx$

$\int -e^{u} \, du$

$-e^{u} + C$

$\boxed{-e^{\cos x} + C}$

12. $\int \frac{e^{\tan x}}{\cos^2 x}$ $\quad u = \tan x$, $du = \sec^2 x \, dx$

$\int e^{u} \, du$

$e^{u} + C$

$\boxed{e^{\tan x} + C}$

13. $\int \frac{\sin x}{\sqrt{\cos x}} \, dx$ $\quad u = \cos x$, $du = -\sin x \, dx$

$\int -u^{-\frac{1}{2}} \, du$

$-2u^{\frac{1}{2}} + C$

$\boxed{-2\sqrt{\cos x} + C}$

14. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} \, dx$ $\quad u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} \, dx$

$\int 2\sec^2 u \, du$

$2\tan u + C$

$\boxed{2\tan\sqrt{x} + C}$

15. $\int 2x \sin x^2 \cos x^2 \, dx$ $\quad u = \sin x^2$, $du = 2x\cos x^2$

$\int u \, du$

$\frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(\sin x^2)^2 + C}$

OR $\quad u = \cos x^2$, $du = -2x \sin x^2 \, dx$

$\int -u \, du = -\frac{1}{2}u^2 + C$

$= \boxed{-\frac{1}{2}(\cos x^2)^2 + C}$