IB Math HL1 Further u-Substitution Name Key Period: ______

$\int x\sqrt{x+2}\,dx$	Let $u = x+2$ Then we can make substitutions replacing x, $x+2$, and dx $u = x+2 \rightarrow du = dx$ and $x = u-2$
$\int x\sqrt{x+2}\,dx$ $= \int (u-2)\sqrt{u}\,du = \int (u^{3/2} - 2u^{1/2})\,du$	Rewrite the integrand in terms of u.
$= \frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}\cdot u^{3/2} + C$	Integrate with respect to u variable. $= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

Example)

1) $\int 2x\sqrt{x+5}\,dx$

$u = x+5 \Rightarrow x = u-5$

$du = dx$

$\int 2(u-5)\sqrt{u}\,du$

$= \int (2u^{3/2} - 10u^{1/2})\,du$

$= (2)\frac{2}{5}u^{5/2} - 10\cdot\frac{2}{3}u^{3/2} + C = \frac{4}{5}(x+5)^{5/2} - \frac{20}{3}(x+5)^{3/2} + C$

2) $\int 2x\sqrt{x^2+5}\,dx$

$u = x^2+5$

$du = 2\cdot x\,dx$

$\int \sqrt{u}\,du = \frac{2}{3}(x^2+5)^{3/2} + C$

3) $\int \frac{3x}{\sqrt{x+1}}\,dx$

$u = x+1 \Rightarrow x = u-1$

$du = dx$

$\int 3(u-1)\cdot u^{-1/2}\cdot du$

$= 3\int (u^{1/2} - u^{-1/2})\,du$

$= \cancel{3}\left(\frac{2}{\cancel{3}}u^{3/2} - 2\cdot u^{1/2}\right) + C = 2(x+1)^{3/2} - 6(x+1)^{1/2} + C$

4) $\int \frac{3x}{\sqrt{x^2+1}}\,dx$

$u = x^2+1$

$\frac{1}{2}du = \frac{\cancel{2}}{\cancel{2}}x\,dx$

$\frac{3}{2}\int u^{-1/2}\,du$

$= \frac{3}{\cancel{2}}\cdot\cancel{2}\sqrt{x^2+1} + C$

$= 3\sqrt{x^2+1} + C$

Practice)

4. $\int x\sqrt{x-3}\,dx$

5. $\int x^2\sqrt{x+1}\,dx$

6. $\int \frac{x}{\sqrt{x+4}}\,dx$

7. $\int \frac{6x-9}{(x^2-3x+5)^3}\,dx$

8. $\int \frac{dx}{x\left(\sqrt[3]{\ln x}\right)}$

9. $\int x\sqrt{x-3}\,dx$

10. $\int \frac{2x-5}{4x^2}\,dx$

IB Math 2 Further u-Substitution Name key.

Evaluate each integral.

1. $\int \frac{4x^3}{\sqrt{4x^4+1}}dx$ 2. $\int \frac{\ln(x+1)}{x+1}dx$ 3. $\int x\sqrt{x+2}dx$

Solution to #3: Let $u = x+2$. Then we can make substitutions replacing x, $x+2$, and dx.

$u = x+2 \rightarrow du = dx$ and $x = u-2$

$\int x\sqrt{x+2}dx = \int (u-2)\sqrt{u}du$ Substitute a u expression for every x expression.

$= \int \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right)du$ Rewrite the integrand in a helpful way.

$= \frac{2}{5}u^{\frac{5}{2}} - 2\cdot\frac{2}{3}u^{\frac{3}{2}} + C$ Integrate.

$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$ Substitute.

Evaluate each integral.

4. $\int x\sqrt{x-3}dx$ 5. $\int x^2\sqrt{x+1}dx$ 6. $\int \frac{x}{\sqrt{x+4}}dx$ 7. $\int \frac{6x-9}{(x^2-3x+5)^3}dx$

8. $\int \frac{dx}{x\left(\sqrt[3]{\ln x}\right)}$ 9. $\int x\sqrt{x-3}dx$ 10. $\int \frac{2x-5}{4x^2}dx$

#4 $u = x-3$

$du = dx$

$x = u+3$

$\int (u+3)\sqrt{u}\,du$

$= \int (u^{3/2} + 3u^{1/2})du$

$= \boxed{\frac{2}{5}(u+3)^{5/2} + 2(u+3)^{3/2} + c}$

#5 $x+1=u$

$du = dx$

$x = u-1$

$\int (u-1)^2\sqrt{u}\,du$

$= \int (u^2-2u+1)\sqrt{u}\,du$

$= \int (u^{5/2} - 2u^{3/2} + u^{1/2})du$

$= \boxed{\frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + c}$

#6. $u = x+4 \Rightarrow x = u-4$

$dx = du$

$\int (u-4)u^{-1/2}du$

$= \int (u^{1/2} - 4u^{-1/2})du$

$= \boxed{\frac{2}{3}(x+4)^{3/2} - 8(x+4)^{1/2} + c}$

#7. $u = x^2-3x+5$

$du = (2x-3)dx$

$\int \frac{6x-9}{x^2-3x+5}dx = \int \frac{3\,du}{u}$

$= \boxed{3\ln|x^2-3x+5| + c}$

#8. $\ln x = u$

$du = \frac{1}{x}dx$

$\int \frac{du}{\sqrt[3]{u}} = \int u^{-1/3}du$

$= \boxed{\frac{3}{2}(\ln x)^{2/3} + c}$

#9. $u = x-3 \Rightarrow x = u+3$.

$du = dx$

$\int (u+3)\sqrt{u}\,du$

$= \int (u^{3/2} + 3u^{1/2})du$

$= \boxed{\frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + c}$

#10. $\int \left(\frac{2x}{4x^2} - \frac{5}{4x^2}\right)dx$

$= \int \left(\frac{1}{2}x^{-1} - \frac{5}{4}x^{-2}\right)dx = \boxed{\frac{1}{2}\ln|x| + \frac{5}{4x} + c}$