Integration by Parts Day 1 Name____________________

Warm Up :

1. Given $f'(x) = x + \frac{d}{dx}(g(x))$, write $f(x)$ in terms of g.

$dx\left(\frac{df}{dx}\right) = \left(x + \frac{d}{dx} g(x)\right) dx$

$= df(x) = x \cdot dx + d\,g(x)$

$\int df(x) = \int x\,dx + \int d\,g(x)$

$\Rightarrow \boxed{f(x) = \frac{1}{2}x^2 + g(x) + C}$

2. a. Find $\frac{d}{dx}(x \sin x)$.

$\Rightarrow \sin x + x \cdot \cos x$

b. Hence, evaluate $\int x \cos x\,dx$.

$= x \sin x + \cos x + C$.

Check: $\frac{d}{dx}(x \sin x + \cos x + C)$

$= \sin x + x\cos x - \sin x = x \cos x$

3. a. Find $\frac{d}{dx}(x \ln x)$.

$\ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$.

b. Hence, evaluate $\int \ln x\,dx$.

$x \cdot \ln x - x + C$

Check: $(x \ln x - x + C)' = \ln x + \cancel{x \cdot \frac{1}{x}} - 1$

$= \ln x$

The Product Rule

$u' \cdot v + u \cdot v'$

$dx\left[\frac{d}{dx}(uv)\right] = \left[\left(\frac{d}{dx}u\right) \cdot v + u\left(\frac{d}{dx}v\right)\right] dx$

$\int d\,uv = \int (du \cdot v + u \cdot dv)$. $\Rightarrow$ $u \cdot v = \int v \cdot du + \int u \cdot dv$.

Integration by Parts

$\int uv'\,dx = \boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$

Examples)

1) $\int xe^{-x}dx$

$u = x$	$dv = e^{-x}dx$
$du = dx$	$v = -e^{-x}$

$= -x \cdot e^{-x} - \int -e^{-x}dx$

$= -xe^{-x} + \int e^{-x}dx$

$= -xe^{-x} - e^{-x} + C$

2) $\int x \cos x\,dx$

$u = x$	$dv = \cos x\,dx$
$du = dx$	$v = \sin x$

$\Rightarrow x \sin x - \int \sin x\,dx$.

$= \boxed{x \sin x + \cos x + C}$

3) $\int \ln x\,dx$

$u = \ln x$	$dv = dx$
$du = \frac{1}{x}dx$	$v = x$

$\Rightarrow x \ln x - \int \frac{1}{x} \cdot x\,dx$

$= x \ln x - x + C$

Integration by Parts Practice 1

Identify u and dv for finding the integral using integration by parts. (Do not evaluate the integral.)

1. $\int x^2 e^{2x} dx$
2. $\int (\ln x)^2 \, dx$
3. $\int x \sec^2 x dx$
4. $\int x^2 \cos x dx$

1: $u = x^2$, $dv = e^{2x} \cdot dx$.

2: $u = (\ln x)^2$, $dv = dx$.

3: $u = x$, $dv = \sec^2 x dx$

4: $u = x^2$, $dv = \cos x dx$.

Evaluate the integral using integration by parts with the given choices of u and dv.

7. $\int x^3 \ln x dx; u = \ln x, dv = x^3 dx$
8. $\int (4x+7)e^x dx; u = 4x+7, dv = e^x dx$
9. $\int x \sin 3x dx; u = x, dv = \sin 3x dx$

7:

$u = \ln x$	$dv = x^3 dx$
$du = \frac{1}{x} dx$	$v = \frac{1}{4}x^4$

$\Rightarrow \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^3 dx$

$= \boxed{\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C}$

8:

$u = 4x+7$	$dv = e^x dx$
$du = 4dx$	$v = e^x$

$= e^x(4x+7) - 4\int e^x dx$

$= e^x(4x+7) - 4e^x + C$

9:

$u = x$	$dv = \sin 3x dx$
$du = dx$	$v = -\frac{1}{3}\cos 3x$

$= -\frac{1}{3}x \cos 3x + \frac{1}{3}\int \cos 3x dx$

$= -\frac{1}{3}x \cos 3x + \frac{1}{9}\sin 3x + C$

Evaluate each integral using integration by parts.

8. $\int x^2 \ln x dx$
9. $\int x \cos 2x dx$
10. $\int x \ln x dx$

#8

$u = \ln x$	$dv = x^2$
$du = \frac{1}{x} dx$	$v = \frac{1}{3}x^3 dx$

$\Rightarrow \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

#9

$u = x$	$dv = \cos 2x dx$
$du = dx$	$v = \frac{1}{2}\sin 2x$

$= \frac{1}{2}x \sin 2x - \frac{1}{2}\int \sin 2x dx$

$= \boxed{\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C}$

#10

$u = \ln x$	$dv = x dx$
$du = \frac{1}{x} dx$	$v = \frac{1}{2}x^2$

$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$