Integration by Parts Day 1

Name Key

Warm Up :

1. Given $f'(x) = x + \frac{d}{dx}(g(x))$, write $f(x)$ in terms of g.

$f'(x) = \frac{df}{dx} = (x + \frac{d}{dx} g(x))\,dx$

$\int df = \int (x\,dx) + \int d\,g(x)$

$\boxed{f(x) = \frac{1}{2}x^2 + g(x)}$

2. a. Find $\frac{d}{dx}(x\sin x)$. b. Hence, evaluate $\int x\cos x\,dx$.

$\frac{d}{dx}(x \cdot \sin x)$

$= x' \cdot \sin x + x \cdot (\sin x)' = \boxed{\sin x + x\cos x}$

$= x\sin x + \cos x + C$

check: $\sin x + x\cos x - \sin x$

$= x\cos x$

3. a. Find $\frac{d}{dx}(x\ln x)$. b. Hence, evaluate $\int \ln x\,dx$.

$= x' \cdot \ln x + x \cdot (\ln x)'$

$= \ln x + x \cdot \frac{1}{x} = \boxed{\ln x + 1}$

$= \boxed{x\ln x - x + C}$

check: $\ln x + x \cdot \frac{1}{x} - 1$

$= \ln x + 1 - 1 = \ln x$

The Product Rule

Integration by Parts

$u' \cdot v + u \cdot v'$

$dx \cdot \frac{d}{dx}(uv) = \left(\frac{d}{dx}u\right) \cdot v + u\left(\frac{dv}{dx}\right)$

$\int uv'\,dx =$

$\int u \cdot \frac{dv}{dx} \cdot dx = \boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$

$\int d(u \cdot v) = \int (du \cdot v + u \cdot dv) \Rightarrow u \cdot v = \int v\,du + \boxed{\int u \cdot dv}$

Examples)

1) $\int xe^{-x}dx$

$\int u \cdot dv$

$u = x$	$dv = e^{-x}dx$
$du = 1dx$	$v = -e^{-x}$

$\oplus$ $\ominus\int$

$= -xe^{-x} + \int +e^{-x}dx$

$= \boxed{-xe^{-x} - e^{-x} + C}$

2) $\int x\cos x\,dx$

$\int u \cdot dv$

$u = x$	$\int dv = \int \cos x\,dx$
$du = dx$	$v = \sin x$

$\Rightarrow x\sin x - \int \sin x\,dx$

$= \boxed{x\sin x + \cos x + C}$

3) $\int \ln x\,dx$

$\int u \cdot dv$

$u = \ln x$	$dv = dx$
$du = \frac{1}{x}dx$	$v = x$

$\oplus$ $\ominus\int$

$\Rightarrow x\ln x - \int x \cdot \frac{1}{x}\,dx$

$= x\ln x - \int 1\,dx$

$= \boxed{x\ln x - x + C}$

Integration by Parts Practice 1

Identify u and dv for finding the integral using integration by parts. (Do not evaluate the integral.)

1. $\int x^2 e^{2x} dx$
2. $\int (\ln x)^2 \, dx$
3. $\int x \sec^2 x dx$
4. $\int x^2 \cos x dx$

Evaluate the integral using integration by parts with the given choices of u and dv.

7. $\int x^3 \ln x dx; u = \ln x, dv = x^3 dx$
8. $\int (4x+7)e^x dx; u = 4x+7, dv = e^x dx$
9. $\int x \sin 3x dx; u = x, dv = \sin 3x dx$

Evaluate each integral using integration by parts.

8. $\int x^2 \ln x dx$
9. $\int x \cos 2x dx$
10. $\int x \ln x dx$

#2. $\int (\ln x)^2 dx$

$\int u \cdot dv$.

$$\boxed{\begin{array}{ll} u = (\ln x)^2 & dv = dx \\ du = 2(\ln x)\cdot\frac{1}{x}dx & v = x \end{array}}$$

$\ominus \int$

$= x(\ln x)^2 - \int 2(\ln x)\frac{1}{\cancel{x}}\cdot\cancel{x}dx$

$= x(\ln x)^2 - 2\int(\ln x)dx$.

$$\boxed{\begin{array}{ll} u = \ln x & dv = dx \\ du = \frac{1}{x}dx & v = x \end{array}}$$

$\ominus \int$

$= x(\ln x)^2 - 2\left[x\ln x - \int dx\right]$

$= \boxed{x(\ln x)^2 - 2x\ln x + 2x + C}$

Integration by Parts Practice 1

1. $u = x^2$
   $dv = e^{2x}\,dx$

2. $u = (\ln x)^2$
   $dv = dx$

3. $u = x$
   $dv = \sec^2 x\,dx$

4. $u = x^2$
   $dv = \cos x\,dx$

5. $\int x^3 \ln x\,dx$ $\quad u = \ln x \quad dv = x^3$
   $du = \frac{1}{x}\,dx \quad v = \frac{1}{4}x^4$

$(\ln x)(\frac{1}{4}x^4) - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$

$\frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^3\,dx$

$\boxed{\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C}$

6. $\int (4x+7)e^x\,dx$ $\quad u = 4x+7 \quad dv = e^x\,dx$
   $du = 4\,dx \quad v = e^x$

$(4x+7)e^x - \int e^x \cdot 4\,dx$

$\boxed{(4x+7)e^x - 4e^x + C}$

7. $\int x \sin 3x\,dx$ $\quad u = x \quad dv = \sin 3x\,dx$
   $du = dx \quad v = -\frac{1}{3}\cos 3x$

$-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,dx$

$\boxed{-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C}$

8. $\int x^2 \ln x \, dx$

$u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{3}x^3$

$\ln x \cdot \frac{1}{3}x^3 - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$

$\frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\,dx$

$\boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$

9. $\int x \cos 2x \, dx$

$u = x \qquad dv = \cos 2x\,dx$

$du = dx \qquad v = \frac{1}{2}\sin 2x$

$x \cdot \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x\,dx$

$\boxed{\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C}$

10. $\int x \ln x \, dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{2}x^2$

$\ln x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$

$\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx$

$\boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$