practice WS #1. Solutions

b. by u-Substitution

1. $\int x\sqrt{9+x}\,dx$

$u = 9+x \rightarrow x = u-9.$

$du = dx$

$\int (u-9)\sqrt{u}\,du$

$= \int (u^{\frac{3}{2}} - 9u^{\frac{1}{2}})\,du.$

$= \frac{2}{5}(9+x)^{\frac{5}{2}} - (\cancel{9}^{3})(\frac{2}{\cancel{3}})(9+x)^{\frac{3}{2}} + c$

$= \boxed{\frac{2}{5}(9+x)^{\frac{5}{2}} - 6(9+x)^{\frac{3}{2}} + c}$

a. by parts

$\int x\sqrt{9+x}\,dx$

$u = x$	$dv = \sqrt{9+x}\,dx$
$du = dx$	$v = \frac{2}{3}(9+x)^{3/2}$

$\ominus$

$\frac{2}{3}x(9+x)^{3/2} - \int \frac{2}{3}(9+x)^{\frac{3}{2}}\,dx$

$= \frac{2}{3}x(9+x)^{3/2} - (\frac{2}{3})(\frac{2}{5})(9+x)^{\frac{5}{2}} + c$

$= \boxed{\frac{2}{3}x(9+x)^{3/2} - \frac{4}{15}(9+x)^{\frac{5}{2}} + c}$

b. by - u-substitution.

#2. $\int \frac{x^3}{\sqrt{4+x^2}}\,dx.$

$u = 4+x^2 \Rightarrow x^2 = u-4$

$du = 2x\,dx. \Rightarrow \frac{1}{2}du = x\,dx$

$\int \frac{(u-4)\frac{1}{2}du}{\sqrt{u}}$

$= \frac{1}{2}\int (u-4)u^{-\frac{1}{2}}\,du$

$= \frac{1}{2}\int (u^{\frac{1}{2}} - 4u^{-\frac{1}{2}})\,du$

$= \frac{1}{2}\left[\frac{2}{3}(4+x^2)^{\frac{3}{2}} - 8(4+x^2)^{\frac{1}{2}}\right] + c. = \boxed{\frac{1}{3}(4+x^2)^{\frac{3}{2}} - 4(4+x^2)^{\frac{1}{2}} + c}$

a. by parts.

$u = x^2$	$dv = \frac{x}{\sqrt{4+x^2}}\,dx$ $u = 4+x^2$, $du = 2x\,dx$, $x(4+x^2)^{-\frac{1}{2}}$
$du = 2x\,dx$	$v = 2(4+x^2)^{\frac{1}{2}}$

$= 2x^2(4+x^2)^{\frac{1}{2}} - \int 4x(4+x^2)^{\frac{1}{2}}\,dx.$

$u = 4+x^2$

$du = 2x\,dx$

$= 2x^2(4+x^2)^{\frac{1}{2}} - (2)(\frac{2}{3})(4+x^2)^{\frac{3}{2}} + c$

$= \boxed{2x^2(4+x^2)^{\frac{1}{2}} - \frac{4}{3}(4+x^2)^{\frac{3}{2}} + c}$

\#3 $\int x^2 \sin x \, dx$.

$$= \boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$$

Tabular method
when repeat Integration by parts.

Alt. Signs	Derivatives u	Antiderivatives dv
+	x^2	$\sin x$.
−	$2x$	$-\cos x$
+	2	$-\sin x$
−	0	$\cos x$

\#4. $\int 4 \arccos x \, dx$.

$u = \arccos x$	$dv = dx$
$du = \frac{-1}{\sqrt{1-x^2}} dx$	$v = x$

$$= 4\int \arccos x \, dx$$

$$= 4\left[x \arccos x\right] - 4\int \frac{-x}{\sqrt{1-x^2}} dx. \leftarrow \text{u-Sub}$$

$u = 1-x^2$

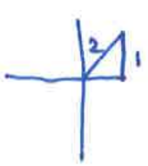
$du = -2x\,dx$.

$\frac{1}{2} du = -x\,dx$.

$$= 4x\arccos x - 4\left(\tfrac{1}{2}\right)\cancel{(2)}(1-x^2)^{\frac{1}{2}} + C$$

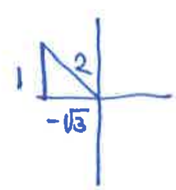
$$= \boxed{4x\arccos x - 4(1-x^2)^{\frac{1}{2}} + C}$$

\#5. a. $\arcsin\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$.



$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

b. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $[0, \pi]$



$= \boxed{\frac{5\pi}{6}}$

c. $\arctan(-1)$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= -\frac{\pi}{4}$