Integration by Parts Practice 2 **Class Copy. Work on your own paper.**

Find each indefinite or definite integral. Solve by the simplest method—not all require integration by parts.

1. $\int \frac{4x}{e^x}dx$
2. $\int \frac{\ln 2x}{x^2}dx$
3. $\int \frac{1}{x(\ln x)^3}dx$
4. $\int \arctan x dx$
5. ~~$\int \theta \sec\theta \tan\theta d\theta$~~
6. $\int x^2 e^{x^3} dx$
7. $\int x^2 e^{-x} dx$
8. $\int_0^3 x e^{\frac{x}{2}} dx$
9. $\int_0^\pi x \sin 2x dx$
10. $\int_0^1 x \arcsin x^2 dx$
11. Integrate $\int x\sqrt{4-x}dx$ a. by parts, letting $dv = \sqrt{4-x}dx$. b. by substitution, letting $u = 4-x$.

Integration by Parts Practice 2

1. $\int \frac{4x}{e^x}\,dx$ $\quad u = 4x \quad dv = e^{-x}\,dx$

$du = 4\,dx \quad v = -e^{-x}$

$-4xe^{-x} - \int -4e^{-x}\,dx$

$\boxed{-4xe^{-x} - 4e^{-x} + C}$

2. $\int \frac{\ln 2x}{x^2}\,dx$ $\quad u = \ln 2x \quad dv = x^{-2}\,dx$

$du = \frac{2}{2x}\,dx \quad v = -x^{-1}$

$-\frac{1}{x}\ln 2x - \int \frac{1}{x} \cdot \frac{-1}{x}\,dx$

$-\frac{1}{x}\ln 2x + \int x^{-2}\,dx$

$\boxed{-\frac{1}{x}\ln 2x - \frac{1}{x} + C}$

3. $\int \frac{1}{x(\ln x)^3}\,dx$ $\quad u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^{-3}\,du$

$-\frac{1}{2}u^{-2} + C = \boxed{\frac{-1}{2(\ln x)^2} + C}$

4. $\int \arctan x\,dx$ $\quad u = \arctan x \quad dv = dx$

$du = \frac{1}{1+x^2}\,dx \quad v = x$

$x\arctan x - \int x \cdot \frac{1}{1+x^2}\,dx$ $\quad w = 1 + x^2$

$dw = 2x\,dx$

$x\arctan x - \int \frac{1}{2} \cdot \frac{1}{w}\,dw$

$x\arctan x - \frac{1}{2}\ln w + C$

$\boxed{x\arctan x - \frac{1}{2}\ln(1+x^2) + C}$

5. $\int \theta \sec\theta \tan\theta \, d\theta$ $\quad u = \theta$ $\quad dv = \sec\theta \tan\theta \, d\theta$

$du = d\theta$ $\quad v = \sec\theta$

$\theta \sec\theta - \int \sec\theta \, d\theta$

Don't do #5.

6. $\int x^2 e^{x^3} dx$ $\quad u = x^3$

$du = 3x^2 dx$

$\int \frac{1}{3} e^u \, du$

$\frac{1}{3} e^u + C = \boxed{\frac{1}{3} e^{x^3} + C}$

7. $\int x^2 e^{-x} dx$ $\quad u = x^2$ $\quad dv = e^{-x} dx$

$du = 2x\,dx$ $\quad v = -e^{-x}$

$-x^2 e^{-x} + \int 2x e^{-x} dx$ $\quad u = 2x$ $\quad dv = e^{-x} dx$

$du = 2dx$ $\quad v = -e^{-x}$

$-x^2 e^{-x} + \left[-2xe^{-x} - \int -2e^{-x} dx\right]$

$\boxed{-x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + C}$

8. $\int_0^3 x e^{\frac{x}{2}} dx$ $\quad u = x$ $\quad dv = e^{\frac{x}{2}}$

$du = dx$ $\quad v = 2e^{\frac{x}{2}}$

$\int x e^{\frac{x}{2}} dx = 2xe^{\frac{x}{2}} - \int 2e^{\frac{x}{2}} dx = 2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} + C$

$\int_0^3 x e^{\frac{x}{2}} dx = \left[2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}}\right]_0^3$

$= \left(6e^{\frac{3}{2}} - 4e^{\frac{3}{2}}\right) - \left(0 - 4e^0\right)$

$= \boxed{2e^{3/2} - 4}$

9. $\int_0^{\pi} x \sin 2x \, dx$

$u = x \qquad dv = \sin 2x \, dx$

$du = dx \qquad v = -\frac{1}{2}\cos 2x$

$$\int x \sin 2x \, dx = -\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x \, dx$$

$$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$$

$$\int_0^{\pi} x \sin 2x \, dx = \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi}$$

$$= \left(-\frac{1}{2}\pi\cos 2\pi + \frac{1}{4}\sin 2\pi\right) - (0 + 0)$$

$$= \boxed{-\frac{\pi}{2}}$$

10. $\int_0^1 x \arcsin x^2 \, dx$

$u = x^2$

$du = 2x \, dx$

$\int_0^1 \frac{1}{2}\arcsin u \, du$

$w = \arcsin u \qquad dv = \frac{1}{2}du$

$dw = \frac{1}{\sqrt{1-u^2}}\cancel{dx} \qquad v = \frac{1}{2}u$

$$\frac{1}{2}u\arcsin u - \int \frac{1}{2}u \cdot \frac{1}{\sqrt{1-u^2}} \, du$$

$$\frac{1}{2}u\arcsin u - \left[-\frac{3}{8}(1-u^2)^{3/2}\right]$$

$$\left[\frac{1}{2}u\arcsin u + \frac{3}{8}(1-u^2)^{3/2}\right]_0^1$$

$$\left(\frac{1}{2}\cdot 1\cdot\arcsin 1 + \frac{3}{8}(0)\right) - \left(0 + \frac{3}{8}\cdot 1\right)$$

$$\boxed{\frac{\pi}{4} - \frac{3}{8}}$$

11. $\int x\sqrt{4-x}\,dx$

a) $u = x$ $\qquad dv = \sqrt{4-x}\,dx$

$du = dx$ $\qquad v = -\frac{2}{3}(4-x)^{3/2}$

$$\int x\sqrt{4-x}\,dx = -\frac{2}{3}x(4-x)^{3/2} - \int -\frac{2}{3}(4-x)^{3/2}\,dx$$

$$= -\frac{2}{3}x(4-x)^{\frac{3}{2}} - \frac{4}{15}(4-x)^{\frac{5}{2}} + C$$

b. $u = 4-x$

$du = -dx$

$$\int x\sqrt{4-x}\,dx = \int -(4-u)\sqrt{u}\,du$$

$$= \int \left(-4u^{\frac{1}{2}} + u^{\frac{3}{2}}\right)du$$

$$= -\frac{8}{3}u^{\frac{3}{2}} + \frac{2}{5}u^{\frac{5}{2}} + C$$

$$= -\frac{8}{3}(4-x)^{\frac{3}{2}} + \frac{2}{5}(4-x)^{\frac{5}{2}} + C$$