

Integration Review 1A (8 questions)

[answers included]

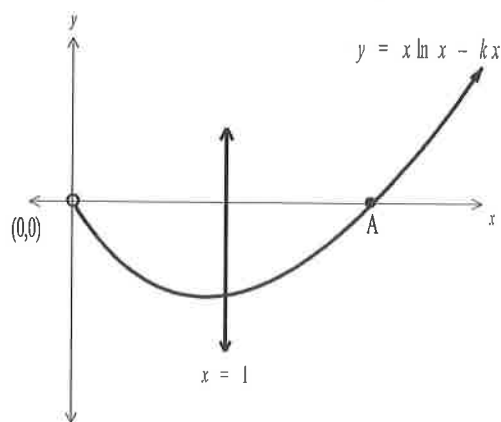
◆ no calculator allowed on these questions ◆

1. By using a suitable substitution, find each of the following:

$$(a) \int \frac{3x}{\sqrt{1-x^2}} dx \qquad (b) \int \sin x \cos x dx$$

2. Use the method of integration by parts to find the following indefinite integrals:

$$(a) \int x e^x dx \qquad (b) \int x \ln x dx$$

3. An object moves in a straight line. At time t seconds the object's velocity is given by $v(t) = t \sin t$. When $t = 0$ the displacement of the object is zero metres.(a) Find an expression for the displacement s in terms of t .(b) What is the total **displacement** of the object from $t = 0$ to $t = 2\pi$. Give **exact** answer.(c) What is the total **distance** traveled by the object from $t = 0$ to $t = 2\pi$. Give **exact** answer.4. Find the exact area of the region enclosed by the curves $y = 3e^x - 2$ and $y = e^{2x}$.5. (a) Find $\int \frac{1}{x+3} dx$. (b) Given that $\int_0^m \frac{1}{x+3} dx = 1$, calculate the exact value of m .6. Find the exact value of the definite integral $\int_1^e \frac{\ln x}{x^2} dx$.7. Consider the function $f(x) = x \ln x - kx$, where k is a constant such that $k \geq 1$.(a) The function has an x -intercept at the point A. Find the x -coordinate of A in terms of k .(b) Find the area of the region bounded by the line $x = 1$, the function f and the x -axis in terms of k .8. Find the exact volume of the solid formed by revolving the region bounded by $y = \frac{6}{x}$, $x = 2$ and $x = 4$ about the x -axis.

Integration Review 1A (8 questions)**Answers**

1. (a) $-3\sqrt{1-x^2} + C$ (b) $-\frac{1}{2}\cos^2 x + C$
2. (a) $xe^x - e^x + C$ (b) $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$
3. (a) $s(t) = \sin t - t \cos t$ (b) -2π metres (c) 4π metres
4. area = $2 \ln 2 - \frac{3}{2}$ units² $\left[\text{OR } \ln 4 - \frac{3}{2} \right]$
5. (a) $\ln|x+3| + C$ (b) $3e-3$
6. $1 - \frac{2}{e}$
7. (a) $A(e^k, 0)$ (b) $\frac{e^{2k} - 2k - 1}{4}$ units²
8. 9π units³