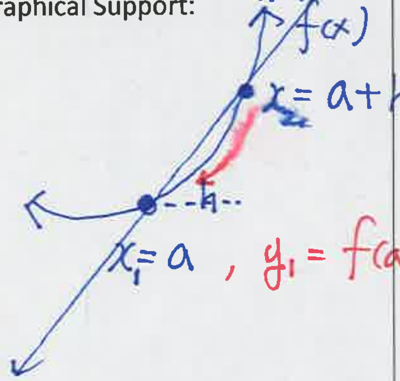
IB Math HL1: *Difference Quotient and Definition of Derivative (the first Principle)*

Slope of the secant Line:

Algebraically: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\frac{f(a+h)-f(a)}{(\cancel{a}+h)-\cancel{a}} = \frac{f(a+h)-f(a)}{h}$$

Graphical Support:

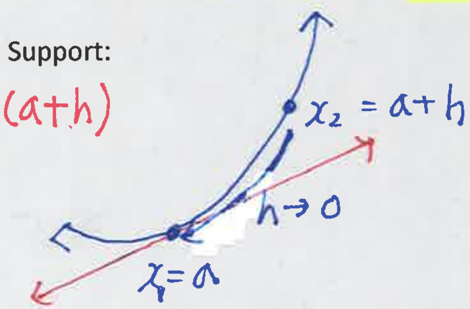


Slope of the tangent Line at $x = a$:
(instantaneous Rate of change at $x = a$)

Algebraically:

$$\left.\frac{dy}{dx}\right|_{x=a} = f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{(\cancel{a}+h)-\cancel{a}} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Graphical Support:

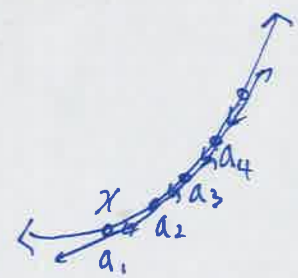


Derivative of (x) : Instantaneous Rate of Change of f(x)

Algebraically (The first Principle) :

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Graphical Support:



Example 1) Find the slope of tangent line at x= 2 for $f(x) = 4x^2 - 3$.

a= 2

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0}\frac{[4(2+h)^2-3]-[4(2)^2-3]}{h}$$

$$= \lim_{h\to 0}\frac{4((2)^2+4h+h^2)\cancel{-3}-4(2)^2\cancel{+3}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{4(2)^2}+16h+4h^2-\cancel{4(2)^2}}{h}$$

$$\lim_{h\to 0}\frac{\cancel{h}(16+4h)}{\cancel{h}}$$

$$= 16 + (4)(0)$$

$$= \boxed{16}$$

Practice) a) Find $f'(2)$ for $f(x) = 2x^2 - 1$

b) Find $f'(x)$ for $f(x) = \sqrt{3x}$

a) $\left.\dfrac{dy}{dx}\right|_{x=2} = 8$

b) $f'(x) = \dfrac{3}{2\sqrt{3x}}$ OR $\dfrac{\sqrt{3x}}{2x}$

Example 2) Find the derivative of $f(x) = \sqrt{x}$ using the first principle.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})}{h}\cdot\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0}\frac{(\cancel{x}+\cancel{h}^{1}-\cancel{x})}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$= \frac{\sqrt{x}}{2x}$$