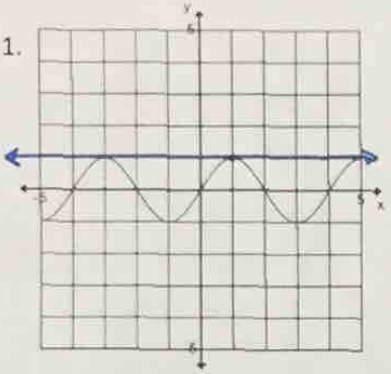
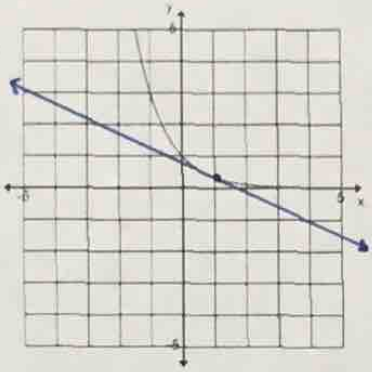
IB Math 2 Intro to Derivatives Exploration

Part 1: Tangent Lines. **Goal: Approximate the slope of a tangent line using a graph.**

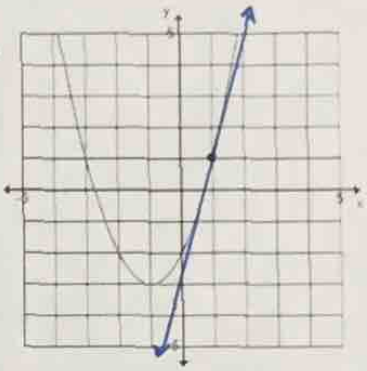
Use a straight edge to draw the line tangent to the each function at x = 1. Use your line to estimate the slope of the function at x = 1.

1. 

$m = 0$

2. 

$m \approx -\frac{1}{2}$

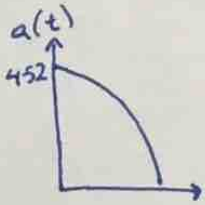
3. 

$m \approx 4$

Part 2: Instantaneous ~~speed~~ velocity. **Goal: Find the instantaneous ~~speed~~ velocity of a falling object.**

In a BASE jumping competition form the Petronas Towers in Kuala Lumpur, the altitude of a professional jumper in the first 3 seconds is given by $a(t) = 452 - 4.8t^2$ meters, where $0 \le t \le 3$ seconds.

1. What will the graph of the altitude of the jumper in the first 3 seconds look like?



2. Does the jumper travel with constant ~~speed~~ velocity? No

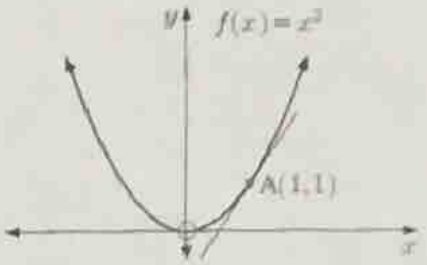
3. Complete the table.

Time c	Change in time from t = c to t = 2 seconds	Change in altitude from t = c to t = 2 seconds	Average ~~speed~~ velocity from t = c to t = 2 seconds
1	1 sec	$432.8 - 447.2 = -14.4$	$\frac{-14.4}{1\text{ sec}} = -14.4$ m/sec
1.5	.5	$432.8 - 441.2 = -8.4$	$\frac{-8.4\text{m}}{.5\text{ sec}} = -16.8$ m/sec
1.8	.2	$432.8 - 436.4 = -3.6$	$\frac{-3.6\text{m}}{.2\text{sec}} = -18$ m/sec
1.9	.1	$432.8 - 434.7 = -1.87$	$\frac{-1.87\text{m}}{.1\text{ sec}} = -18.7$ m/sec
1.99	.01	$432.8 - 432.9915 = .19152$	$\frac{-.19152}{.01} = -19.152$ m/sec
1.999	.001	$432.8 - 432.819 = -.0191952$	$\frac{-.0191952}{.001} = -19.1952$ m/sec

4. What do you suspect is the ~~speed~~ velocity of the jumper at t = 2 seconds?

≈ -19.2 m/sec

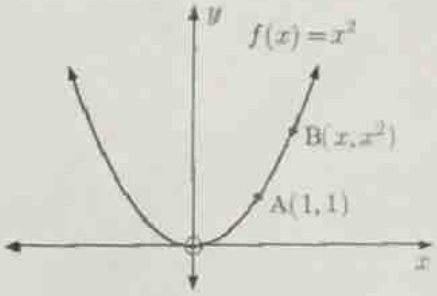
Part 3: The slope of a tangent. Goal: Determine the slope of a tangent line.



Given a curve $f(x)$, we wish to find the slope (gradient) of the tangent at the point $(a, f(a))$. For example, the point $A(1,1)$ lies on the curve $f(x)=x^2$. By the end of this investigation, we will know the slope of the tangent at A.

1. Suppose B lies on $f(x)=x^2$ and B has coordinates (x,x^2).

   a. Show that the chord $\overline{AB}$ has slope $\frac{f(x)-f(1)}{x-1}$ or $\frac{x^2-1}{x-1}$.



$m = \frac{y_2-y_1}{x_2-x_1} = \frac{x^2-1}{x-1}$

   b. Complete the table.

x	Point	Slope of $\overline{AB}$
5	(5, 25)	6
3	(3,9)	4
2	(2,4)	3
1.5	(1.5, 2.25)	2.5
1.1	(1.1, 1.21)	2.1
1.01	(1.01, 1.0201)	2.01
1.001	(1.001, 1.002001)	2.001

$\frac{25-1}{5-1} = \frac{24}{4} = 6$

$\frac{9-1}{3-1} = \frac{8}{2} = 4$

   c. Comment on the slope of $\overline{AB}$ as x gets closer to 1.

The slope of $\overline{AB}$ approaches 2

2. What do you suspect is the slope of the tangent at A? 2

3. Challenge: Write a mathematical expression that represents the process we completed in problem 1. Evaluate your expression and compare your answer with #3.

$$\lim_{x\to 1} \frac{x^2-1}{x-1}$$

$$\lim_{x\to 1} \frac{(x-1)(x+1)}{x-1}$$

$$\lim_{x\to 1} (x+1)$$

$$1+1$$

$$\boxed{2}$$