Name: ______________ Group Members: ______________

Exploration Introduction to Limits

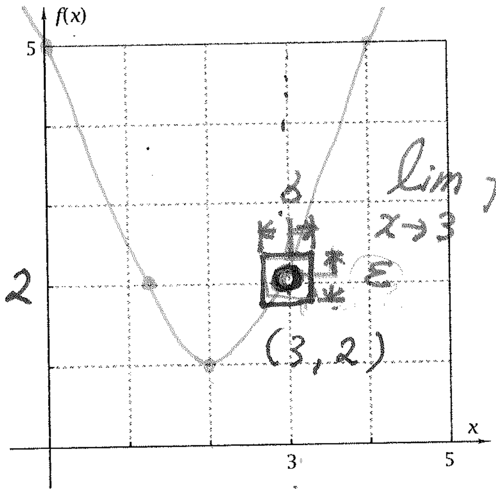
Date: ______________

Objective: Find the limit of a function that approaches an indeterminate form at a particular value of x and relate it to the definition.

1. Plot on your grapher the graph of this function.

$$f(x) = \frac{x^3 - 7x^2 + 17x - 15}{x - 3}$$

Use a friendly window with $x = 3$ as a grid point, but with the grid turned off. Sketch the results here. Show the behavior of the function in a neighborhood of $x = 3$.



$\lim\limits_{x \to 3} f(x) = 2.$

2. Substitute 3 for x in the equation for $f(x)$. What form does the answer take? What name is given to an expression of this form? $f(3) = \frac{27-63+51-15}{3-3} = \boxed{\frac{0}{0}}$

undefined. ✓

3. The graph of f has a **removable discontinuity** at $x = 3$. The y-value at this discontinuity is the **limit** of $f(x)$ as x approaches 3. What number does this limit equal?

Appeared to be '2'.

4. Make a table of values of $f(x)$ for each 0.1 unit change in x-value from 2.5 through 3.5.

x	$f(x)$
2.5	1.25 ←
2.6	1.36
2.7	1.49
2.8	1.64
2.9	1.81
3.0	? Error ≈ ⇒ 2.
3.1	2.21
3.2	2.44
3.3	2.69
3.4	2.96
3.5	3.25 ←

5. Between what two numbers does $f(x)$ stay when x is kept in the open interval (2.5, 3.5)?

f(x) stays between 1.25 and 3.25.

6. Simplify the fraction for $f(x)$. Solve numerically to find the two numbers close to 3 between which x must be kept if $f(x)$ is to stay between 1.99 and 2.01. solve.

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 17 & -15 \\ & & 3 & -12 & 15 \\ \hline & 1 & -4 & 5 & 0 \end{array} \Rightarrow Q: (x^2-4x+5) = 1.99$$

$$f(x) = \frac{(x-3)(x^2-4x+5)}{(x-3)} = x^2-4x+5 = 2.01$$

solve.

$x = 3 \Rightarrow 2.$

7. How far from $x = 3$ (to the left and to the right) are the two x-values in Problem 6?

$2.99498743 < x < 3.00498756$

$1.99 < f(x) < 2.01$

$0.995 < x < 1.005$

8. For the statement "If x is within 0.0049875 units of 3 (but not equal to 3), then $f(x)$ is within 0.01 unit of 2," write the largest number that can go in the blank.

9. The formal definition of limit is

$L = \lim\limits_{x \to c} f(x)$ if and only if

- for any positive number ε (no matter how small)
- there is a positive number δ such that
- if x is within δ units of c, but not equal to c,
- then $f(x)$ is within ε units of L.

The four numbers L, c, ε, and δ all appear in Problem 8. Which is which?

$L = 2$
$c = 3$
$\varepsilon = 0.01$
$\delta = 0.0049875$

10. What did you learn as a result of doing this Exploration that you did not know before?