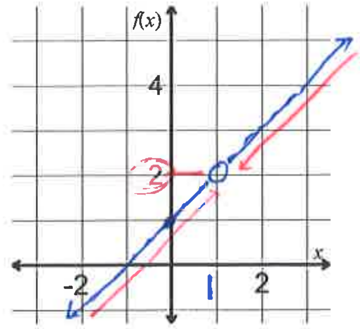
Notes.

IB Math HL 1 Introduction to Limits

1. Warm up:

a. On your GFC, plot the graph of $f(x)=\dfrac{x^2-1}{x-1}$

Sketch the result.




b. Complete the table.

| $x$ | 0.8 | 0.9 | 0.99 | 0.999 | 1 | 1.001 | 1.01 | 1.1 | 1.2 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.8 | 1.9 | 1.99 | 1.999 | undef | 2.001 | 2.01 | 2.1 | 2.2 |

c. Connecting with previous knowledge: When you substitute x = 1, what form does the answer take? What name is given to an expression of this form?

$f(1)=\dfrac{1^2-1}{1-1}=\dfrac{0}{0}=$ undef.

Connecting with new concept: The graph of f has a hole / Removable discontinuity at x = 1. The f value at this discontinuity is the **limit** of f(x) as x approaches 1. So in calculus, you will be asked "What is the **limit** of $f(x)$ as $x$ approaches 1?"

Calculus notation is $\lim\limits_{x\to 1} f(x)=$ 2 :The limit of f(x) as x approaches 1 is 2.

**Informal Definition of Limit #1:**

What y-value does $f(x)$ get close to as $x$ approaches c?

Close: as close as you need to be convinced of the result

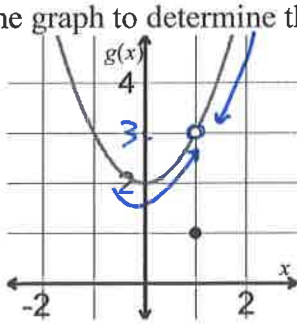
Approaches: closer and closer but not actually there*

*This means that, for the purpose of this limit, we don't care what f(c) is or if it exists.

Notation: $\lim\limits_{x\to c} f(x)=$ ____

2. Use the graph to determine the indicated limit.

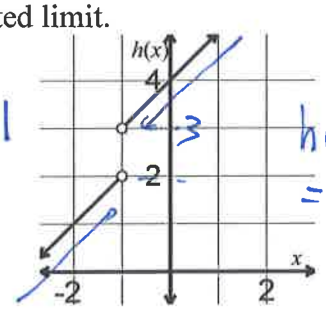
a.




g(1) = 1

$\lim\limits_{x\to 1} g(x)=$ 3

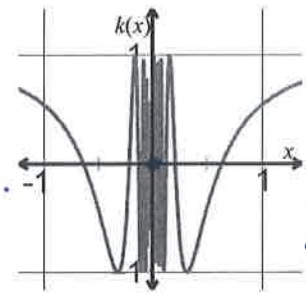
b.




h(-1) = undef.

$\lim\limits_{x\to -1} h(x)=$ 2 OR 3 = DNE

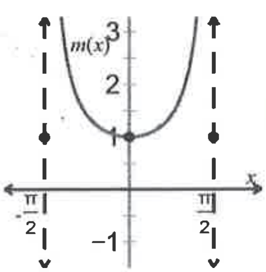
c.




k(0) = undef.

$\lim\limits_{x\to 0} k(x)=$ DNE.

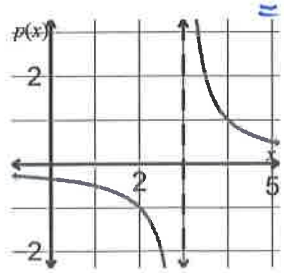
d.




m(0) = 1

$\lim\limits_{x\to 0} m(x)=$ 1

e.




P(3) = undef.

$\lim\limits_{x\to 3} p(x)=$ $\infty$ OR $-\infty$

= DNE (undef.)

3. Use the table values to determine the limit.

a. $\lim_{x\to 2}\frac{x^2+x-2}{x+2} =$ 1

| $x$ | 1.997 | 1.998 | 1.999 | 2 | 2.001 | 2.002 | 2.003 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.997 | 0.998 | 0.999 | 1 | 1.001 | 1.002 | 1.003 |

b. $\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2} =$ 0.25

| $x$ | 1.997 | 1.998 | 1.999 | 2 | 2.001 | 2.002 | 2.003 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.25004 | 0.25003 | 0.25001 | undef. | 0.24998 | 0.24996 | 0.24995 |

between 0.25001 and 0.24998

c. $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} =$ 0.25

| $x$ | 3.997 | 3.998 | 3.999 | 4 | 4.001 | 4.002 | 4.003 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | .25004 | .25003 | 0.25001 | undef | 0.24998 | 0.24996 | 0.24995 |

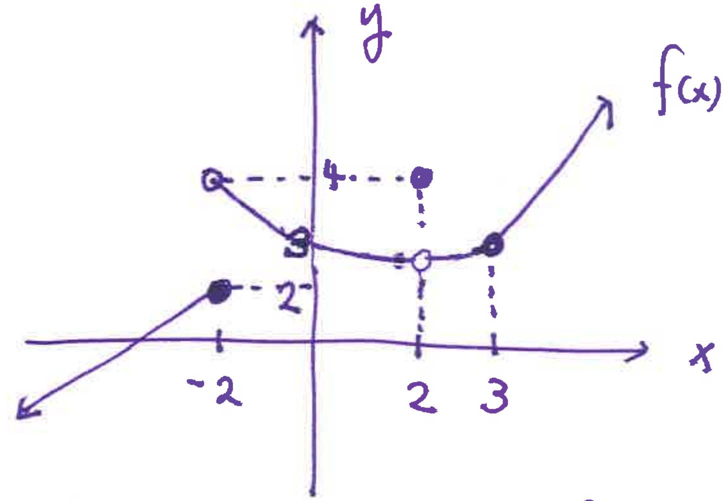
Between 0.25001 and 0.24998

d. $\lim_{x\to 0}\frac{\sin x}{x} =$ 1

| $x$ (radians) | -.3 | -.2 | -.1 | 0 | .1 | .2 | .3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.98506 | 0.99334 | 0.99833 | undef. | 0.99833 | 0.99334 | 0.98506 |

Between 0.99833 and 0.99833.

Exit slip




1) $f(3) = 3$ $\qquad \lim_{x \to 3} f(x) = 3$

2) $f(2) = 4$ $\qquad \lim_{x \to 2} f(x) = 3$

3) $f(-2) = 2$ $\qquad \lim_{x \to -2} f(x) = \text{DNE}$