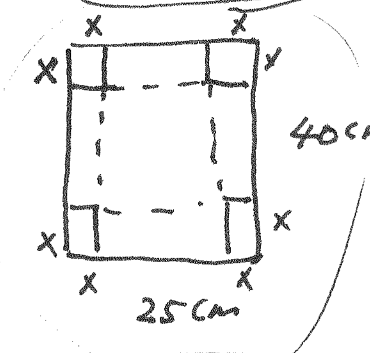
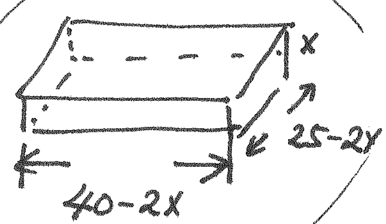
**IB Math 2: Introduction to Optimization Problem Solving** **Name:** key

**Strategy:**

1. **Draw a diagram of given situation with appropriate notation.**
2. **Construct a formula with the variable to be optimized as the subject.**
   **(Remember to write in one variable using the given restriction)**
3. **Find the first derivative and solve for x which make the first derivative zero.**
4. **Confirm if the solution is maximum or minimum and revisit if the solution is reasonable.**

1) A rectangular cake dish is make by cutting out squares from the corners of a 25 cm by 40 cm rectangle of tin-plate, and then folding the metal to form the container. What size square must be out t to product the cake dish of maximum volume?




Volume = W · L · H

$V(x) = (40-2x)(25-2x)\cdot x$

$\boxed{0 < x < 12.5}$

$= \cancel{x2x}$

$= 4x^3 - 50x^2 - 80x^2 + 1000x$

Solve $\frac{dV}{dx} = 0$.

$V(x) = 4x^3 - 130x^2 + 1000x$

$\frac{dV}{dx} = 12x^2 - 260x + 1000$

$= 4(3x^2 - 65x + 250)$

$= 4(3x - 50)(x - 5) = 0$

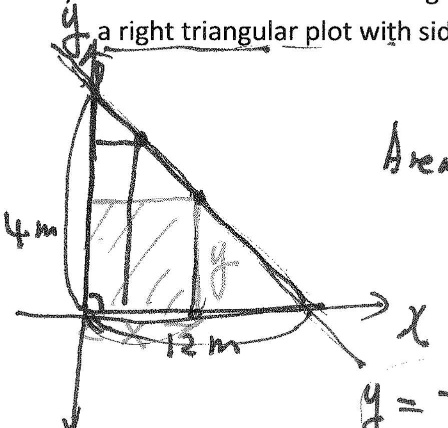
$\cancel{x = \frac{50}{3}}$ $\boxed{x = 5}$.

Check $V'' = 24x - 260$

$V''(5) < 0$ Max.

$\therefore$ $\boxed{5\text{cm by } 5\text{cm}}$

2) You need to fence a rectangular play zone for children. What is maximum area for the play zone if it is to fit into a right triangular plot with sides measuring 4 m and 12 m?




Area = x · y.

$y = -\frac{4}{12}x + 4 = -\frac{1}{3}x + 4$

$A(x) = x\left(-\frac{1}{3}x + 4\right) = -\frac{1}{3}x^2 + 4x$

$\frac{dA}{dx} = -\frac{2}{3}x + 4 = 0$

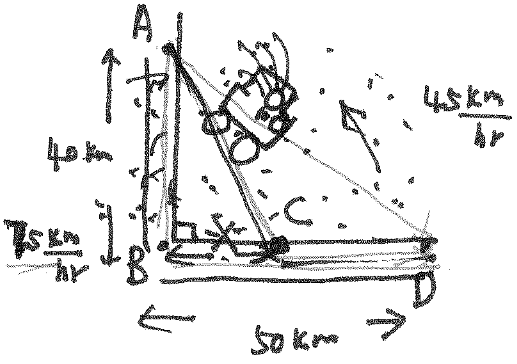
$+\frac{2}{3}x = +4$ $x = 4\left(\frac{3}{2}\right)$

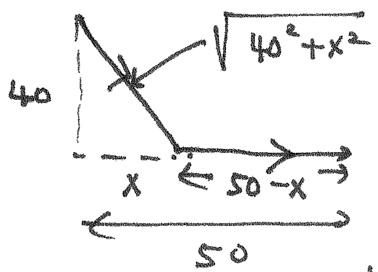
$x = 6$ m

$y = -\frac{1}{3}(6) + 4 = 2$ m

Area = 2×6 = $\boxed{12\ m^2}$

3) A dune buggy is on the desert at point A located 40 km from a point B, which lies on a long straight road, as shown. The driver can travel at 45 km/hr on the desert and 75 km/hr on the road. The driver will win a prize if she arrives at the finish line at point D, 50 km from B, in 84 min or less. What route should she travel to minimize the time of travel? Does she win the prize?




$\boxed{x = 30\text{ km}}$

$$\Rightarrow \text{Distance} = \sqrt{40^2+x^2} + (50-x)$$

$$\boxed{\text{Time} = \left(\frac{1}{45}\right)\left(\sqrt{40^2+x^2}\right) + \left(\frac{1}{75}\right)(50-x)} \approx 83\text{ min}$$

$$\frac{dT}{dx} = \left(\frac{1}{45}\right)\left(\frac{1}{2}\right)(40^2+x^2)^{-\frac{1}{2}}(2x) + \left(\frac{1}{75}\right)(-1) = 0$$

$$\Rightarrow \frac{x}{45\sqrt{40^2+x^2}} = \frac{1}{75}$$

$$\Rightarrow \left(75x = 45\sqrt{40^2+x^2}\right) \div 15$$

$$\Rightarrow (5x)^2 = \left(3\sqrt{40^2+x^2}\right)^2$$

$$\Rightarrow 25x^2 = 9(40^2+x^2)$$

$$\Rightarrow 25x^2 = 9\cdot 40^2 + 9x^2$$

$$-9x^2 \qquad -9x^2$$

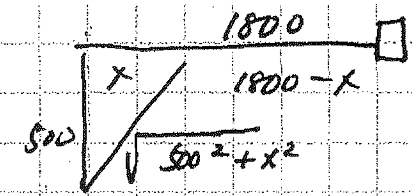
$$\Rightarrow \frac{16x^2}{16} = \frac{9\cdot 40^2}{16}$$

$$x = \sqrt{\frac{9\cdot 40^2}{16}} = \frac{3\cdot 40}{4} = \boxed{30}$$

4) A swimmer is at point 500 m from the closest point on a straight shoreline. She needs to reach a cottage located 1800 m down shore from the closest point. If she swims at 4 m/s and she walks at 6 m/s, how far from the cottage should she come ashore so as to arrive at the cottage in the shortest time?

Key

#4



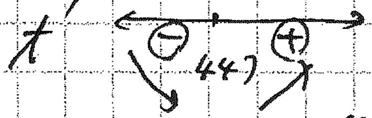

$$t = \frac{\sqrt{500^2+x^2}}{4} + \frac{1800-x}{6}$$

$$t' = \left(\frac{1}{4}\right)\left(\frac{1}{\cancel{2}}\right)(\cancel{2}x)\left(500^2+x^2\right)^{-\frac{1}{2}} - \frac{1}{6} = 0$$

$$\frac{x}{4\sqrt{500^2+x^2}} = \frac{1}{6}. \Rightarrow \overset{3}{\cancel{6}}x = \overset{2}{\cancel{4}}\sqrt{500^2+x^2}$$

$$\frac{9x^2}{4} = 500^2 + x^2$$

$$\boxed{x = 447.\ m}$$




min.

Sign diagram.

$$\Rightarrow t = \frac{\sqrt{500^2+447^2}}{4} + \frac{1800-447}{6}.$$