IB Math HL1 : Kinematics Day one Name: Key Period: ______

Motion in a straight Line.

- Distance: (S(t): The position of the object on the line is a function of time, t.
- Velocity: $v(t)=\frac{ds}{dt}=\lim_{h\to 0}\frac{s(t+h)-s(t)}{h}$: The rate of change of S(t) with respect to time.

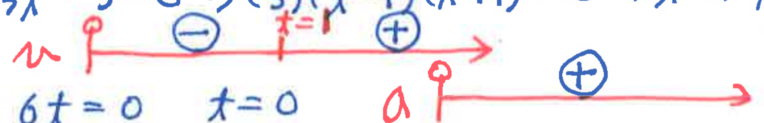
  Average Velocity: $\frac{s(t_2)-s(t_1)}{t_2-t_1}$ Speed: $|v(t)|$
- Acceleration: $a(t)=\frac{dv}{dt}=\frac{d^2s}{dt^2}=\lim_{h\to 0}\frac{v(t+h)-v(t)}{h}$: The rate of change of v(t) with respect to time.

Example 1) A particle moves in straight lien with position relative to 0 given by $s(t)=t^3-3t+1$.

a. Find the velocity function and acceleration function, and draw sign diagrams for each. $t \ge 0$

$\Rightarrow v(t)=\frac{ds}{dt}=3t^2-3 \Rightarrow 3t^2-3=0 \Rightarrow (3)(t-1)(t+1)=0 \Rightarrow t=1$ $\cancel{t=-1}$

$\Rightarrow a(t)=\frac{dv}{dt}=6t \Rightarrow 6t=0$ $t=0$



b. Find the initial conditions when t=0 and describe the motion at this instant.

$t=0$ where $S(0)=1$ ← left → Right

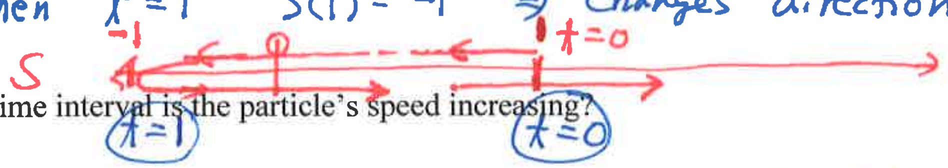
Velocity $v(0)=-3$ Moving left.

acceleration $a(0)=0$ Not accelerating

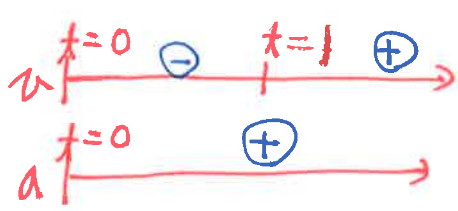
c. Find the position of the particle when changes in direction occur. Hence draw a motion diagram for the particle.

$v(t)=3t^2-3=0 \Rightarrow 3(t-1)(t+1)=0 \Rightarrow t=1$, $\cancel{t=-1}$

when $t=1$ $S(1)=-1$ ⇒ Changes direction.



f. For what time interval is the particle's speed increasing?



Speed increasing $(1, \infty)$

Speed decreasing $(0, 1)$

$t=0$: v ⊖, $a=0$

Notes: *The speed is increasing where the signs of velocity and acceleration are same.*
The speed is decreasing where the signs of velocity and acceleration are opposite.

Example 2) Assume that the position at time t (in seconds) of an object moving along a line is given by $s(t) = 3t^3 - 40.5t^2 + 162t$ on [0, 8].

a) Find the initial position, velocity, and acceleration for the object and discuss the motion.

$t=0$

$S(t) = 3t^3 - 40.5t^2 + 162t$ $\qquad S(0) = 0$: position $S=0$

$v(t) = 9t^2 - 81t + 162$ $\qquad v(0) = 162$: Moves to Right

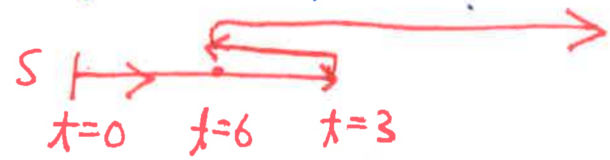
$a(t) = 18t - 81$ $\qquad a(0) = -81$: Decelerating.

b) Find the time when the object changes in direction.

$$v(t) = 9t^2 - 81t + 162 = 0$$
$$= 9(t^2 - 9t + 18) = 0$$
$$= 9(t-6)(t-3) = 0$$

$\Rightarrow$ The object changes direction at $t=3$ and $t=6$.

c) Compute the total distance traveled.



$[0, 8]$

$S(0)=0$ $\qquad S(3) = 202.5$

$S(6) = 162$ $\qquad S(8) = 240$.

$$S_{total} = (202.5) + |202.5 - 162| + |162 - 240|$$
$$= 321 \text{ unit}$$