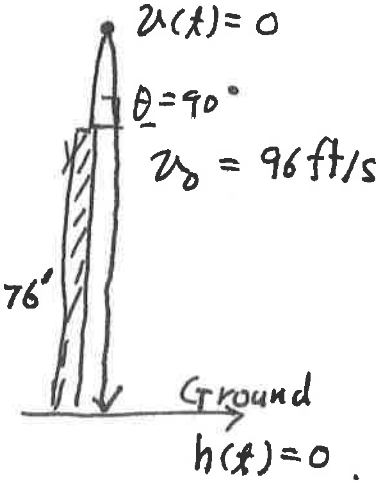
IB Math HL1 : Kinematics    Day two    Name: ____________________    Period: ______

**Example 3)** Falling Body Problem $h(t) = -16t^2 + (v_0 \sin\theta)t + h_0$
where $v_0$ is the initial velocity and $h_0$ is the initial height.

Suppose a person standing at the top of the Tower of Pisa (176ft high) throws a ball directly upward with an initial speed of 96ft/s.

a. Find the ball's height, its velocity, and acceleration at time t.




- $h(t) = -16t^2 + 96t + 176$.
- $v(t) = \frac{dh}{dt} = -32t + 96$.  • $a(t) = -32$

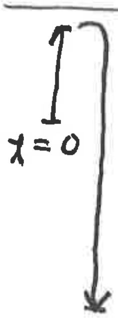
b. When does the ball hit the ground, and what is its impact velocity?

$h(t) = -16t^2 + 96t + 176 = 0$

$t = \cancel{-1.47}$    $t = 7.47$

$v(7.47) = -32(7.47) + 96 \approx \boxed{-143 \text{ ft/s}}$

c. How far does the ball travel during its flight?




$v(t) = -32t + 96 = 0 \Rightarrow t = 3$ sec

Total flight: $|h(0) - h(3)| + |h(3) - h(7.47)|$

$= |176 - 320| + |320 - 0| = \boxed{464 \text{ ft}}$

$h(3) = 320$

$h(7.47) = 0$

Instantaneous Rate of Change: $\left.\frac{df}{dx}\right|_{x=a}$

Suppose f(x) is differentiable at $x = a$. The instantaneous rate of change of f(x) with resp0ect to x at $x = a$ is the value of the derivative of f(x) at $x = a$.

Example 4) Water is draining from a swimming pool. The remaining volume of water after t minutes is $V = 200(50 - t^2)$ cubic meter.

a) Find the average water leavings the pool in the first 5 minutes.

$\text{Average} = \frac{V(5) - V(0)}{5 - 0} = \frac{5000 - 10{,}000}{5} = \boxed{-1000 \ m^3/s}$.

b) Find the instantaneous rate at which the water is leaving at t=5 minutes.

$\frac{dv}{dt} = -200(2t)$

$\left.\frac{dv}{dt}\right|_{t=5} = (-200)(2 \cdot 5) = \boxed{-2000 \ m^3/s}$