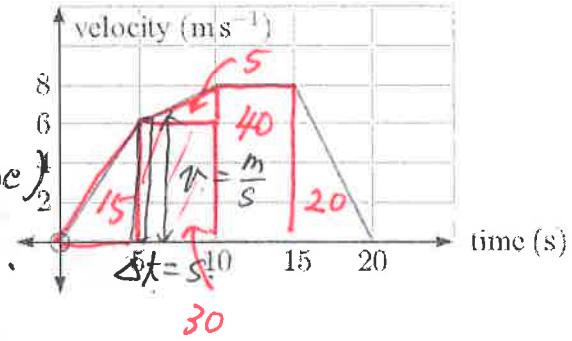
22C Kinematics

1. A runner has the velocity-time graph shown.
Find the total distance traveled by the runner.

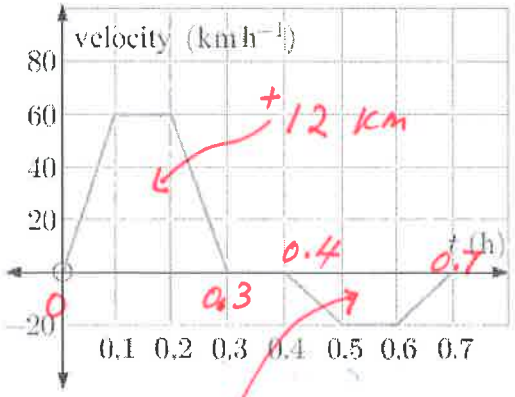



distance = (velocity)(time)

unit = $(\frac{m}{s})(s) = m$.

Total distance : **110 m**

2. A car travels along a straight road with the velocity-time function, $v(t)$, illustrated.




a. What is the significance of the graph:
i. above the $t$-axis?

Move in the positive direction.

ii. below the $t$-axis?

Move in the negative direction.

b. Find the total distance travelled by the car.

$S_{total} = 12 + 4 =$ **16 km**

c. Find the final displacement of the car from its starting point.

$S_{distancement} = 12 - 4 =$ **8 km**

d. Represent parts b and c using integrals.

b. $S_{\text{total distance}} = \int_0^{0.3} v(t)\,dt - \int_{0.4}^{0.7} v(t)\,dt = \int_0^{0.7} |v(t)|\,dt$.

c. $S_{displacement} = \int_0^{0.3} v(t)\,dt + \int_{0.4}^{0.7} v(t)\,dt = \int_0^{0.7} v(t)\,dt$

| Using differential Calculus | | | | Using integral Calculus |
|---|---|---|---|---|
| Position function | Velocity function | Acceleration function | Speed | Displacement |
| $s(t)$ | $v(t) = \frac{ds}{dt}$ | $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ | $\lvert v(t) \rvert$ | $\Delta s = s(t_f) - s(t_i) = \int_{t_i}^{t_f} v(t)\,dt$ |
| Total Distance Traveled | | | | Total Distance Traveled: $s_{total} = \int_{t_i}^{t_f} \lvert v(t) \rvert\,dt$ |

3. A particle P moves in a straight line with velocity function $v(t) = t^2 - 3t + 2$ m/sec, $t \geq 0$.

a. If the particle's initial position is at 7, write an equation for the particle's position at any time $t \geq 0$.

$t = 0,\ s = 7$.

$$s(t) = \int (t^2 - 3t + 2)\,dt = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t + C$$

$$s(0) = 7 = 0 + C \Rightarrow C = 7$$

$$s(t) = \boxed{\frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t + 7}$$

b. Find the displacement of P after 4 seconds.

$s_{displacement} = s(4) - s(0)$

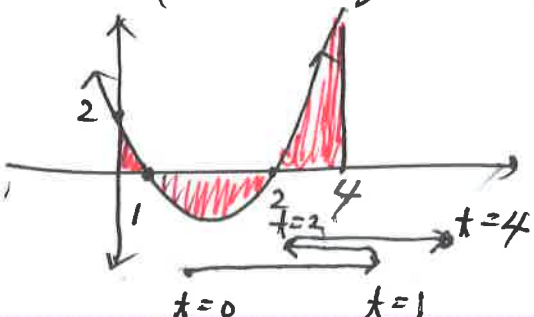
$$\int_0^4 (t^2 - 3t + 2)\,dt = \left[\frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t\right]_0^4$$

$$= \frac{1}{3}(4)^3 - \frac{3}{2}(4)^2 + 2(4) = \boxed{\frac{16}{3}}$$

Total distance

c. How far does P travel in the first 4 seconds of motion?

$v(t) = (t^2 - 3t + 2) = (t-2)(t-1)$




$$s_{total} = \int_0^1 (t^2 - 3t + 2)\,dt - \int_1^2 (t^2 - 3t + 2)\,dt + \int_2^4 (t^2 - 3t + 2)\,dt = \int_0^4 \lvert t^2 - 3t + 2 \rvert\,dt$$

$= 5.\overline{6}$ m