LEVEL 1

These questions are done by making use of the standard antidifferentiation rules, and so we only provide answers to these:

1. i. $\frac{2}{5}\sqrt{x^5}+x^2+c$ **ii.** $ax-bx^3+c$ **iii.** $\frac{4}{3}\sqrt{x^3}-\frac{\sqrt{2}}{2}x^2+2\sqrt{x}+c$ **iv.** $-2\cos\left(\frac{1}{2}x\right)+\frac{1}{2}e^{-2x}+c$

v. $4\ln x+\frac{1}{3}\tan 3x+c$ **vi.** $-e^{2-x}+\ln(x-2)+c$ **vii.** $\frac{2}{9}\sqrt{(3x-2)^3}+c$ **viii.** $-\frac{1}{5}(2-5x)^4+c$

ix. constant (c) (tricky!) **x.** $2\ln(2x+3)-4e^{-\frac{1}{2}x}+c$ **xi.** $\frac{1}{3}x^3-4x-\frac{4}{x}+c$

xii. $-a\ln(a-x)+\frac{1}{a-1}\times\frac{1}{(a-x)^{a-1}}+c$ **2. i.** $\int\left(x^2+2-\frac{1}{x}\right)dx=\frac{1}{3}x^3+2x-\ln x+c$

ii. $\int\left(1-\frac{2}{t}+\frac{1}{t^2}\right)dt=t-2\ln t-\frac{1}{t}+c$ **iii.** $\frac{1}{2}x^4-3x^3+\frac{9}{2}x^2+c$ **iv.** $\frac{5}{2}\sqrt{4x+2}+c$

v. $\frac{1}{2}\ln(6x+1)+c$ **vi.** $\int(x^{1/2}-2x+x^{3/2})dx=\frac{2}{3}x^{3/2}-x^2+\frac{2}{5}x^{5/2}+c$

vii. $-\cos\left(2x-\frac{\pi}{2}\right)+c$ **viii.** $-\frac{3}{4}(5-x)^{4/3}+c$ **ix.** $-\frac{1}{3}e^{-3x}-\frac{1}{\pi}\sin(\pi x)+c$ **x.** $\frac{1}{4}\tan(4x)-4x+c$

xi. $\frac{1}{e}\sin(ex)+e^x+c$ **xii.** $\int(e^{2x}-2+e^{-2x})dx=\frac{1}{2}e^{2x}-2x-\frac{1}{2}e^{-2x}+c$

3. i. $\frac{1}{16}(2x+1)^8+c$ **ii.** $\frac{1}{6}(4x-3)^{3/2}+c$ **iii.** $\frac{1}{8}(5-4x)^{-2}+c$ **iv.** $\frac{1}{12}(9x+2)^{4/3}+c$

4. i. $\ln(2x+1)+c$ **ii.** $-3\ln(1-x)+c$ **iii.** $\frac{1}{2}\ln(8x+5)+c$ **iv.** $-\frac{2}{x-1}+c$

5. i. $\left[-\frac{1}{9}(2-3x)^3\right]_0^1=1$ **ii.** $\left[\frac{5}{8}\ln(8x-5)\right]_1^2=\frac{5}{8}ln\left(\frac{11}{3}\right)$ **iii.** $\left[-\frac{1}{12}(2-8x)^{3/2}\right]_0^{-2}=-\frac{13}{3}\sqrt{2}$

6. i. $-2\cos\left(\frac{1}{2}x-1\right)+c$ **ii.** $\cos(3-x)+c$ **iii.** $-\sin x+c$ **iv.** $\frac{1}{2}\tan(2x+1)+c$

v. $\int\tan^2(2-x)dx=\int(\sec^2(2-x)-1)dx=-\tan(2-x)-x+c$ **vi.** $\frac{1}{\sqrt{3}}\sin(\sqrt{3}x-1)+c$

LEVEL 2

1. i. $\frac{du}{dx}=-2x\therefore\int 2x\sqrt{4-x^2}dx=\int-\sqrt{u}du=-\frac{2}{3}\sqrt{u^3}+c=-\frac{2}{3}\sqrt{(4-x^2)^3}+c$

ii. $\int\left(\frac{u-4}{2}\right)\sqrt{u}\times\frac{1}{2}du=\frac{1}{4}\int(u^{3/2}-4u^{1/2})du=\frac{1}{4}\left(\frac{2}{5}u^{5/2}-\frac{8}{3}u^{3/2}\right)+c$

$=\frac{1}{4}\left(\frac{2}{5}(2x+4)^{5/2}-\frac{8}{3}(2x+4)^{3/2}\right)+c$ **iii.** $\int\frac{3}{u}\times\frac{1}{2}du=\frac{3}{2}\ln(u)+c=\frac{3}{2}\ln(t^2+9)+c$

iv. $\int 2u^2du=\frac{2}{3}u^3+c=\frac{2}{3}(\ln x)^3+c$ **v.** $\int u^3\times-1du=-\frac{1}{4}u^4+c=-\frac{1}{4}\cos^4x+c$

vi. $\int e^u du=e^u+c=e^{\sin x}+c$ **2. i.** $e^{x^2+1}+c$ **ii.** $\frac{1}{6}(x^3+x)^6+c$ **iii.** $\frac{1}{2}\sin(2-x^2)+c$

iv. $\frac{1}{2}\times\frac{1}{5}\sin^5 2x+c=\frac{1}{10}\sin^5 2x+c$ **v.** $(1+e^x)^3+c$ **vi.** $-\frac{1}{4}\times\frac{2}{3}(1-2x^2)^{3/2}+c=-\frac{1}{6}(1-2x^2)^{3/2}+c$

vii. $\frac{1}{2}\ln(4-\cos 2x)+c$ **viii.** $-\frac{1}{(x^2+x-2)}+c$ **ix.** $-\ln(1-e^x)+c$

3. a. $\int_1^4(u-1)\sqrt{u}du=\left[\frac{2}{5}u^2\sqrt{u}-\frac{2}{3}u\sqrt{u}\right]_1^4=\frac{116}{15}$ **b.** $\int_1^4\frac{u+2}{\sqrt{u}}du=\left[\frac{2}{3}u\sqrt{u}+4\sqrt{u}\right]_1^4=\frac{26}{3}$