Example 5)

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x \Rightarrow (1)^\infty$$ (Indeterminate form of Limit)

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = L$$ (Suppose the value of limit is L)

① ln both sides

$$\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^{x} = \ln L$$

② Apply the property of log

$$\lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right) = \ln L$$

$$\Rightarrow \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \ln L$$ $\left(\frac{(\ln 1+\frac{1}{\infty})}{\frac{1}{\infty}} = \frac{0}{0} \right.$: Indeterminate form of Limit)

③ Apply L'Hôpital's Rule

$$\lim_{x\to\infty} \frac{\left[\frac{1}{\left(1+\frac{1}{x}\right)}\right]\cdot\left(\cancel{\frac{-1}{x^2}}\right)}{\cancel{\frac{-1}{x^2}}} = \ln L$$

④ Simplify

$$\lim_{x\to\infty} \frac{1}{1+\frac{1}{x}} = \ln L$$

$$\Rightarrow 1 = \ln L \Rightarrow \boxed{L = e}$$

$$\boxed{\therefore \lim_{n\to\infty} \left(1+\frac{1}{x}\right)^x = e}$$

Day Two ②

IB Math 3: Evaluate the limit w.s

Name: ______________

Period: ________

1. $\lim_{x\to1}\frac{x^3-1}{x^2-1}$ $\left(=\frac{0}{0}\right)$

$= \lim_{x\to1} \frac{3x^2}{2x^2} = \boxed{\frac{3}{2}}$

2. $\lim_{x\to3}\frac{x^3-27}{x^2-9}$ $\left(=\frac{0}{0}\right)$

$= \lim_{x\to3} \frac{3x^2}{2x} = \frac{3\cdot(3)^2}{2(3)} = \boxed{\frac{9}{2}}$

3 $\lim_{x\to1}\frac{x^{10}-1}{x-1}$ $\left(=\frac{0}{0}\right)$

$= \lim_{x\to1} \frac{10x^9}{1} = \boxed{10}$

4. $\lim_{x\to0}\frac{1-\cos^2 x}{\sin^3 x}$ $\left(=\frac{0}{0}\right)$

$= \lim_{x\to0} \frac{-2\cancel{\cos x}\cdot(-\cancel{\sin x})}{3\sin^{\cancel{2}}x\cdot(\cancel{\cos x})} = \lim_{x\to0} \frac{2}{3\sin x} = \boxed{\infty}$

5. ~~$\lim_{t\to\pi}\frac{\cos\frac{\pi}{2}}{\pi-x}$~~

6. $\lim_{x\to0}\frac{1-\cos x}{x^2}$ $\left(=\frac{0}{0}\right)$

$= \lim_{x\to0} \frac{\sin x}{2x} \left(=\frac{0}{0}\right) \Rightarrow \lim_{x\to0} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$

7. $\lim_{x\to0}\frac{\sin 3x\sin 2x}{x\sin 4x}$

$= \lim_{x\to0} \frac{\sin 3x}{x} \left(=\frac{0}{0}\right) \cdot \lim_{x\to0} \frac{\sin 2x}{\sin 4x} = \left(\frac{0}{0}\right)$

$= \lim_{x\to0} \frac{3\cos 3x}{1} \cdot \lim_{x\to0} \frac{2\cos 2x}{4\cos 4x}$

$= 3\cdot\frac{1}{2} = \boxed{\frac{3}{2}}$

8. $\lim_{x\to+\infty}(1+\frac{1}{2x})^{3x}$

$= \lim_{x\to\infty} \ln\left(1+\frac{1}{2x}\right)^{3x} = \ln L$

$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{2x}\right)}{\frac{1}{3x}} = \left(\frac{0}{0}\right) = \ln L$

$= \lim_{x\to\infty} \frac{\left(\frac{1}{1+2x}\right)\left(\frac{1}{2}\cdot\frac{-1}{\cancel{x^2}}\right)}{\left(\frac{1}{3}\right)\left(\frac{-1}{\cancel{x^2}}\right)} = \ln L$

$\Rightarrow \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2} = \ln L \Rightarrow \boxed{L = e^{\frac{3}{2}}}$

9. $\lim_{x\to-\infty}(1-\frac{3}{x})^{2x}$

$$\lim_{x\to-\infty} \ln\left(1-\frac{3}{x}\right)^{2x} = \ln L$$

$$= \lim_{x\to-\infty} \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{2x}} \left(=\frac{0}{0}\right) = \ln L$$

$$= \lim_{x\to-\infty} \frac{\left(\frac{1}{1-\cancel{\frac{3}{x}}^{\,0}}\right)\left(\frac{3}{\cancel{x^2}}\right)}{\left(\frac{1}{2}\right)\left(-\frac{1}{\cancel{x^2}}\right)} = \lim_{x\to-\infty} \frac{3}{-\frac{1}{2}} = 6 = \ln L$$

$$\boxed{L = e^6}$$

10. $\lim_{t\to\infty}(e^x+x)^{1/x}$

$$\lim_{t\to\infty} \ln\left(e^x+x\right)^{\frac{1}{x}} = \ln L$$

$$= \lim_{t\to\infty} \frac{1}{x}\ln\left[e^x+x\right] \left(=\frac{\infty}{\infty}\right) = \ln L$$

$$= \lim_{x\to\infty} \frac{\cancel{\left(\frac{1}{e^x+x}\right)}^{\,0}\left(e^x+1\right)}{1} = \ln L$$

$\ln L = \infty$

$$\boxed{L = \infty}$$

11. $\lim_{x\to\infty}(1+\frac{1}{5x})^{-x}$

$$\lim_{x\to\infty} \ln\left(1+\frac{1}{5x}\right)^{-x} = \ln L$$

$$= \lim_{x\to\infty} -x\ln\left(1+\frac{1}{5x}\right) = \ln L$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{5x}\right)}{-\frac{1}{x}} = \left(\frac{0}{0}\right) = \ln L$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{1}{5x}}\right)\left(\frac{1}{5}\left(\frac{-1}{\cancel{x^2}}\right)\right)}{\frac{1}{\cancel{x^2}}} = \lim_{x\to\infty} \left(\frac{1}{1+\frac{1}{5x}}\right)\left(\frac{-1}{5}\right) = \left(\frac{-1}{5}\right) = \ln L \Rightarrow \boxed{L = e^{-\frac{1}{5}}}$$

12. $\lim_{x\to+\infty}(1+x)^{1/x}$

$$\lim_{x\to\infty} \ln\left(1+x\right)^{\frac{1}{x}} = \ln L$$

$$= \lim_{x\to\infty} \frac{1}{x}\ln\left(1+x\right) \left(=\frac{\infty}{\infty}\right) = \ln L$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{1+x}\right)(1)}{1} = \ln L \Rightarrow 1 = \ln L$$

$L = e$

~~13. $\lim_{x\to+\infty}(7-\frac{1}{x})^{6x}$~~

Take #13 Out