WS key

Evaluate the limit

#1. $\lim\limits_{x\to 1} \dfrac{x^3-1}{x^2-1} \left(\frac{0}{0}\right) = \lim\limits_{x\to 1} \dfrac{3x^2}{2x} \left(\frac{0}{0}\right) = \boxed{\dfrac{3}{2}}$

#2. $\lim\limits_{x\to 3} \dfrac{x^3-27}{x^2-9} \left(\frac{0}{0}\right) = \lim\limits_{x\to 3} \dfrac{3x^2}{2x} = \dfrac{27}{6} = \boxed{\dfrac{9}{2}}$.

#3. $\lim\limits_{x\to 1} \dfrac{x^{10}-1}{x-1} \left(\frac{0}{0}\right) = \lim\limits_{x\to 1} \dfrac{10x^9}{1} = \boxed{10}$

#4 $\lim\limits_{x\to 0} \dfrac{1-\cos^2 x}{\sin^3 x} \left(\frac{0}{0}\right) = \lim\limits_{x\to 0} \dfrac{+2\cancel{\cos x}\cdot(+\sin x)}{(3\sin^2 x)(\cancel{\cos x})} = \lim\limits_{x\to 0} \dfrac{2}{3\sin x}$

$\hookrightarrow \lim\limits_{x\to 0} \dfrac{\sin^2 x}{\sin^3 x} = \lim\limits_{x\to 0} \dfrac{1}{\sin x} = \dfrac{1}{0} = \infty$ $\qquad = \dfrac{2}{0} = \boxed{\infty}$

#5 $\lim\limits_{x\to \pi} \dfrac{\cos\frac{x}{2}}{\pi - x} \left(\frac{0}{0}\right) = \lim\limits_{x\to\pi} \dfrac{0}{-1} = 0$

#6 $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} \left(\frac{0}{0}\right) = \lim\limits_{x\to 0} \dfrac{\sin x}{2x} \left(\frac{0}{0}\right) = \lim\limits_{x\to 0} \dfrac{\cos x}{2} = \dfrac{1}{2} = \boxed{\dfrac{1}{2}}$

#7. $\lim\limits_{x\to 0} \dfrac{\sin 3x \sin 2x}{x \sin 4x} = \lim\limits_{x\to 0} \dfrac{\sin 3x}{x} \left(\frac{0}{0}\right) \lim\limits_{x\to 0} \dfrac{\sin 2x}{\sin 4x} \left(\frac{0}{0}\right)$

$= \lim\limits_{x\to 0} \dfrac{3\cos 3x}{1} \lim\limits_{x\to 0} \dfrac{2\cos 2x}{4\cos 4x} = 3 \cdot \dfrac{2}{\cancel{4}\,2} = \boxed{\dfrac{3}{2}}$

#8. $\lim\limits_{x\to\infty} \ln\left(1+\frac{1}{2x}\right)^{3x} = \ln L$

$= \lim\limits_{x\to\infty} \dfrac{\ln\left(1+\frac{1}{2x}\right)}{\frac{1}{3x}} = \ln L \quad \left(\frac{0}{0}\right)$

$= \lim\limits_{x\to\infty} \dfrac{\left(\frac{1}{1+\frac{1}{2x}}\right)\left(\frac{1}{2}\right)\left(\cancel{\frac{-1}{x^2}}\right)}{\left(\frac{+1}{3}\right)\left(\cancel{\frac{-1}{x^2}}\right)} = \dfrac{3}{2} = \ln L \qquad \boxed{L = e^{3/2}}$

#9.

$$\lim_{x\to\infty} \ln\left(1-\frac{3}{x}\right)^{2x} = \ln L$$

$$\lim_{x\to-\infty} \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{2x}} = \ln L \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to-\infty} \frac{\left(\frac{1}{1-\frac{3}{x}}\right)\left(3\right)\left(\frac{1}{\cancel{x^2}}\right)}{\left(-\frac{1}{2}\right)\left(\frac{1}{\cancel{x^2}}\right)} = \ln L$$

$$= \lim_{x\to-\infty}\left(\frac{1}{1-\frac{3}{x}}\right)(-6) = -6 = \ln L$$

$$\boxed{L = e^{-6}}$$

#10.

$$\lim_{x\to\infty} \frac{1}{x}\ln\left(e^{x}+x\right) = \ln L$$

$$= \lim_{x\to\infty} \frac{\ln\left(e^{x}+x\right)}{x} = \ln L \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{\frac{1}{e^{x}+x}\left(e^{x}+1\right)}{1} = \ln L \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{e^{x}}{e^{x}+1} = 1 = \ln L$$

$$\boxed{L = e}$$

#11.

$$\lim_{x\to\infty} -x\ln\left(1+\frac{1}{5x}\right) = \ln L$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{5x}\right)}{\frac{-1}{x}} = \ln L \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{1}{5x}}\right)\left(-\frac{1}{5}\,\frac{1}{\cancel{x^2}}\right)}{\frac{1}{\cancel{x^2}}} = \ln L.$$

$$= -\frac{1}{5} = \ln L \Rightarrow \boxed{L = e^{-\frac{1}{5}}}$$

#12.

$$\lim_{x\to\infty} \ln(1+x)^{\frac{1}{x}} = \ln L$$

$$= \lim_{x\to\infty} \frac{\ln(1+x)}{x} \quad \left(\frac{\infty}{\infty}\right) = \ln L$$

$$= \lim_{x\to\infty}\left(\frac{1}{1+x}\right) = 0 = \ln L.$$

$$\boxed{L = e^{0} = 1}$$

#13.

$$\lim_{x\to\infty} \ln\left(7-\frac{1}{x}\right)^{6x} = \ln L$$

$$\lim_{x\to\infty} \frac{\ln\left(7-\frac{1}{x}\right)}{\frac{1}{6x}} = \frac{\ln 7}{0} = \boxed{\infty} = \ln L.$$

$$\boxed{L = \infty}$$