Limit Review solutions.

#1.

a.

①	②	③
$f(0) = 4$ Not continuous $\lim\limits_{x\to 0} f(x) = DNE$	$f(2) = 3$ Not continuous $\lim\limits_{x\to 2} f(x) \neq f(2)$	$f(4) = 2$ Not continuous. $\lim\limits_{x\to 4} f(x) \neq f(4)$.

b. $\lim\limits_{x\to 2} f(x) = 1$ c. $\lim\limits_{x\to 0} f(x) = DNE$ d. $\lim\limits_{x\to 0^-} f(x) = 1$.

#2. Take it out.

#3 Use calculator

a)

x	1.98	1.99	2	2.01	2.02
y	-.001125	-.00056	undifined	.00056	0.0011

$$\lim_{x\to 2} \frac{x^2-4x+4}{x^3+5x^2-14x} = 0$$

b)

x	-.002	-.001	0	.001	.002
y	-502	-1002	und.	998	498

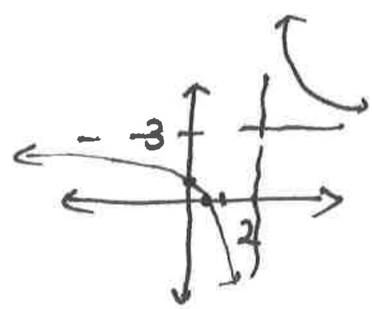
$$\lim_{x\to 0} \frac{1-\sin 2x}{x} = DNE.$$

#4.

a) $$\lim_{x\to 3} \frac{x^2-4x+9}{x^2+x-8} = \frac{3^2-4\cdot 3+9}{3^2+3-8} = \boxed{\frac{3}{2}}$$

b) $$\lim_{x\to 2^-} \frac{3x-1}{x-2} = \boxed{-\infty}.$$

work by graph



OR $$\lim_{x\to 2^-} \frac{3x-1}{x-2} \to \frac{3\cdot 2-1}{1.99-2} = \frac{5}{-.001} = -\infty$$

c) $$\lim_{x\to 0} \frac{\sin 9x}{\sin 5x} = \lim_{x\to 0} \left(\frac{\sin 9x}{9x}\right)\left(\frac{5x}{\sin 5x}\right)\left(\frac{9}{5}\right)$$

$$= 1\cdot 1\cdot \frac{9}{5} = \boxed{\frac{9}{5}}$$

✓ d) $\lim_{x\to\frac{1}{2}^-} \frac{|2x-1|}{2x-1} = \lim_{x\to\frac{1}{2}^-} \frac{-(\cancel{2x-1})}{\cancel{2x-1}} = \lim_{x\to\frac{1}{2}^-} (-1) = -1$

$\leftarrow \frac{|2x-1|}{2x-1}$

for $x < \frac{1}{2}$

$|2x-1| = -(2x-1)$

No cal

e) $\lim_{x\to\infty} \frac{3x+5}{2-7x} \;\frac{\div x}{\div x}\; \lim_{x\to\infty} \frac{3+\frac{5}{x}}{\frac{2}{x}-7} = \frac{3+\frac{5}{\infty}}{\frac{2}{\infty}-7} = \boxed{\frac{3}{-7}}$ OR

H. Asym $\boxed{y = \frac{3}{-7}}$

✓ f) $\lim_{x\to 0} \frac{1-\sin x}{\cos^2 x} = \frac{1-0}{1^2} = 1$

✓ g) $\lim_{x\to 0} \frac{e^{3x}-1}{e^x-1} = \lim_{x\to 0} \frac{(\cancel{e^x-1})(e^{2x}+e^x+1)}{(\cancel{e^x-1})} = \lim_{x\to 0}(e^{2x}+e^x+1)$

$= e^0+e^0+1 = 3$

※ $a^3-b^3 = (a-b)(a^2+ab+b^2)$

$a = e^x,\; b=1 \Rightarrow e^{3x}-1 = (e^x-1)(e^{2x}+e^x+1)$

☆ h) $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim_{x\to 3} \frac{\frac{3}{3x}-\frac{x}{3x}}{x-3} = \lim_{x\to 3} \frac{\frac{3-x}{3x}}{x-3}$

$= \lim_{x\to 3} \frac{\overset{-1}{\cancel{3-x}}}{3x(\cancel{x-3})} = \lim_{x\to 3} \frac{-1}{3x} = \frac{-1}{3(3)} = \frac{-1}{9}$

☆ 5) $\lim_{x\to 1^-} (Ax+3) = A+3 \Rightarrow$ Must be 5 to be continuous.

$A+3=5 \quad A=2$

$\lim_{x\to 1^+} (x^2+B) = 1+B \Rightarrow$ Must be 5 to be continuous

$1+B=5 \quad B=4$

✓ 6)

$\lim_{x\to 0} \frac{(\sqrt{ax+b}-1)(\sqrt{ax+b}+1)}{x\,(\sqrt{ax+b}+1)} = \lim_{x\to 0} \frac{(ax+b-1)}{x(\sqrt{ax+b}+1)}$ Need to be reduced

$= \lim_{x\to 0} \frac{(a\cancel{x})}{\cancel{x}(\sqrt{ax+1}+1)} = \lim_{x\to 0} \frac{a}{\sqrt{ax+1}+1} = \frac{a}{\sqrt{1}+1} = 1$ $\boxed{b=1}$ $\boxed{a=2}$

(7) a. $f(x) = 3x^2 - 7x$

$$f'(x) = \lim_{h \to 0} \frac{[3(x+h)^2 - 7(x+h)] - [3x^2 - 7x]}{h}$$

$$= \lim_{h \to 0} \frac{[\cancel{3x^2} + 6xh + h^2 - \cancel{7x} - 7h - \cancel{3x^2} + \cancel{7x}]}{h}$$

$$= \lim_{h \to 0} \frac{6x\cancel{h} + \cancel{h^2} - 7\cancel{h}}{\cancel{h}} = \lim_{h \to 0} 6x + h - 7 = \boxed{6x - 7}$$

b. $f(x) = \sqrt{3x-1}$

$$f'(x) = \lim_{h \to 0} \frac{(\sqrt{3(x+h)-1} - \sqrt{3x-1})(\sqrt{3(x+h)-1} + \sqrt{3x-1})}{h \, (\sqrt{3(x+h)-1} + \sqrt{3x-1})}$$

$$= \lim_{h \to 0} \frac{[3(x+h) - 1 - (3x-1)]}{h(\sqrt{3(x+h)-1} + \sqrt{3x-1})} = \lim_{h \to 0} \frac{\cancel{3x} + 3h - \cancel{1} - \cancel{3x} + \cancel{1}}{h(\sqrt{3(x+h)-1} + \sqrt{3x-1})}$$

$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}(\sqrt{3(x+h)-1} + \sqrt{3x-1})} = \boxed{\frac{3}{2\sqrt{3x-1}}}$$