Limit Review W.S Solutions.

#1.

a. $f(0) = 4$ Not continuous. $\lim_{x \to 0} f(x) = DNE$

$f(2) = 3$ Not continuous $\lim_{x \to 2} f(x) = 1$

$f(4) = 2$ Not continuous $\lim_{x \to 4} f(x) = \infty$ (DNE)

b. $\lim_{x \to 2} f(x) = 1$

c. $\lim_{x \to 0} f(x) = DNE$

d. $\lim_{x \to 0^-} f(x) = 1$.

#2. at $x = 0$ $\left(\lim_{x \to 0} f(x) = DNE \right)$

at $x = -2$ $\left(\lim_{x \to -2^-} f(x) = -\infty \neq \lim_{x \to 2^+} f(x) = \infty \right)$

$\left(\text{OR } \lim_{x \to -2} f(x) = DNE. \right)$

at $x = 2$ $\lim_{x \to 2} f(x) \neq f(2)$

at $x = 4$ $\lim_{x \to 4} f(x) = \infty \neq f(4) = 2$.

#3. Use calculator

a.

x	1.98	1.99	2	2.01	2.02
y	−0.001125	−0.00056	undef	+0.0056	0.0011

$$\lim_{x\to 2} \frac{x^2-4x+4}{x^3+5x^2-14x} = 0.$$

b.

x	−.002	−.001	0	.001	.002
y	−502	−1002	undef.	998	498

opposite sign with large numbers.

$$\lim_{x\to 0} \frac{1-\sin 2x}{x} = DNE. \quad \left(\text{Appears to be } \lim_{x\to 0^+} f(x) = \infty,\ \lim_{x\to 0^-} f(x) = -\infty\right)$$

#4. a) $\displaystyle\lim_{x\to 3} \frac{x^2-4x+9}{x^2+x-8} = \frac{3^2-4\cdot 3+9}{3^2+3-8} = \frac{3}{2}$

Substitution

c) Use $\displaystyle\lim_{u\to 0}\frac{\sin u}{u} = 1$

$$\lim_{x\to 0}\frac{\sin 9x}{\sin 5x} = \lim_{x\to 0}\left(\frac{\sin 9x}{9x}\cdot\frac{9}{1}\right)\lim_{x\to 0}\left(\frac{5x}{\sin 5x}\right)\left(\frac{1}{5}\right)$$

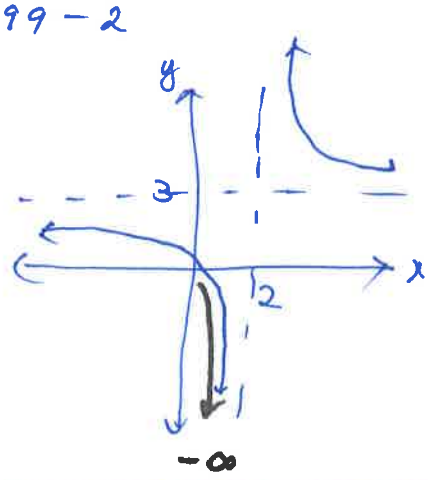
$$= \frac{9}{1}\cdot\frac{1}{5} = \boxed{\frac{9}{5}}$$

b) $\displaystyle\lim_{x\to 2^-}\frac{3x-1}{x-2} = -\infty$

Supporting work. Support with Numerical method. OR Graph.

$$\frac{3(1.999)-1}{1.999-2} = -\infty$$

OR



d) $\lim_{x\to\frac{1}{2}^-} \frac{|2x-1|}{2x-1} = \lim_{x\to\frac{1}{2}^-} \frac{-(\cancel{2x-1})}{(\cancel{2x-1})} = \boxed{-1}$

use a piece wise function.

e) $\lim_{x\to\infty} \frac{3x+5}{2-7x} \begin{matrix}(\div x)\\ = \\ (\div x)\end{matrix} \lim_{x\to\infty} \frac{3+\frac{5}{x}}{\frac{2}{x}-7} = \frac{3}{-7}$

OR. $f(x) = \frac{3x+5}{2-7x} \Rightarrow$ H. Asym: $y = \frac{3}{-7}$. Use a graph.

f) $\lim_{x\to 0} \frac{1-\sin x}{\cos^2 x} = \frac{1-0}{1} = \boxed{1}$

Substitution.

Notes:

$a^3 - b^3 = (a-b)(a^2+ab+1)$

g) $\lim_{x\to 0} \frac{e^{3x}-1}{e^x-1}$ $\left(\frac{0}{0}\right) \Rightarrow$ Reduction method.

$= \lim_{x\to 0} \frac{(\cancel{e^x-1})(e^{2x}+e^x+1)}{(\cancel{e^x-1})} = \lim_{x\to 0} e^{2x}+e^x+1 = e^0+e^0+e^0 = 1+1+1 = \boxed{3}$

h) $\lim_{x\to 3} \frac{\left(\frac{1}{x}-\frac{1}{3}\right)}{(x-3)}$ $\left(\frac{0}{0}\right) \Rightarrow$ Reduction method.

$\lim_{x\to 3} \frac{\left(\frac{1}{x}-\frac{1}{3}\right)\cdot 3x}{(x-3)\cdot 3x} = \lim_{x\to 3} \frac{(\cancel{3-x})^{-1}}{(\cancel{x-3})(3x)} = \lim_{x\to 3} \frac{-1}{3x} = \boxed{\frac{-1}{9}}$

#5.

$$f(x) = \begin{cases} Ax+3 & \text{if } x<1 \\ 5 & \text{if } x=1 \\ x^2+B & \text{if } x>1 \end{cases}$$

$$\Rightarrow \lim_{x\to 1^-} Ax+3 = 5. \Rightarrow A+3=5 \Rightarrow \boxed{A=2}$$

$$\lim_{x\to 1^+} x^2+B=5 \Rightarrow 1+B=5 \Rightarrow \boxed{B=4}$$

#6.

$$\lim_{x\to 0} \frac{(\sqrt{ax+b}-1)(\sqrt{ax+b}+1)}{(x)(\sqrt{ax+b}+1)} = \lim_{x\to 0} \frac{ax+b-1}{x[ax+b+1]} = 1$$

$\frac{ax+b-1}{x} \Rightarrow$ Must be reduced to be $a \Rightarrow \therefore\ b-1=0$ $\boxed{b=1}$

$$\Rightarrow \lim_{x\to 0} \frac{a\cancel{x}}{\cancel{x}[ax+2]} = \lim_{x\to 0} \frac{a}{[ax+2]} = 1$$

$$\frac{a}{2}=1 \qquad \boxed{a=2}$$

#7.

a. $f'(x) = \lim\limits_{h \to 0} \dfrac{[3(x+h)^2 - 7(x+h)] - [3x^2 - 7x]}{h}$

$= \lim\limits_{h \to 0} \dfrac{3(x^2+2hx+h^2 - \cancel{7x} - 7h) - 3x^2 + \cancel{7x}}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{3x^2} + 6hx + 3h^2 - 7h - \cancel{3x^2}}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{h}[6x + 3h - 7]}{\cancel{h}} = \boxed{6x - 7}$

b. $f'(x) = \lim\limits_{h \to 0} \dfrac{[\sqrt{3(x+h)-1} - \sqrt{3x-1}]}{h} \dfrac{[\sqrt{3(x+h)-1} + \sqrt{3x-1}]}{[\sqrt{3(x+h)-1} + \sqrt{3x-1}]}$

$= \lim\limits_{h \to 0} \dfrac{3(\cancel{x}+h) - \cancel{1} - (\cancel{3x} - \cancel{1})}{h[\sqrt{3(x+h)-1} + \sqrt{3x-1}]} = \lim\limits_{h \to 0} \dfrac{3\cancel{h}}{\cancel{h}[\sqrt{3(x+h)-1} + \sqrt{3x-1}]}$

$= \dfrac{3}{\sqrt{3x-1} + \sqrt{3x-1}} = \boxed{\dfrac{3}{2\sqrt{3x-1}}}$ OR $\boxed{\dfrac{3\sqrt{3x-1}}{2(3x-1)}}$