Name: _____ Group Members: _____

Exploration 5-2a: Linear Combination of Cosine and Sine

Date: _____

Objective: Write the linear combination $y = b \cos \theta + c \sin \theta$ as $y = A \cos (\theta - D)$, a sinusoid with a phase displacement.

The expression on the right in the equation

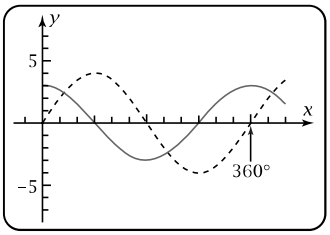
$$y = 3 \cos \theta + 4 \sin \theta$$

is called a **linear combination** of $\cos \theta$ and $\sin \theta$. That is, y equals a constant times cosine, plus a constant times sine. In this Exploration, you will learn how to express such a linear combination as a cosine with a phase displacement.

1. The graph shows

   $y_1 = 3 \cos$ and $y_2 = 4 \sin \theta$

   Which graph is which?



2. Plot y_3 and sketch it on the figure.

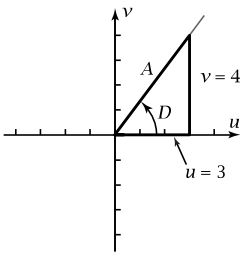
   $y_3 = 3 \cos \theta + 4 \sin \theta$

3. The graph of y_3 is a sinusoid. Find the amplitude A and the phase displacement D using the MAXIMUM feature of your grapher.

   $A =$ _____ $D =$ _____

4. Plot $y_4 = A \cos (\theta - D)$ using Problem 3 results. Does the graph coincide with y_3? _____

5. The uv-diagram here shows an angle with $u = 3$, the coefficient of cosine in y_3, and $v = 4$, the coefficient of sine. Show that the hypotenuse equals A from Problem 3.

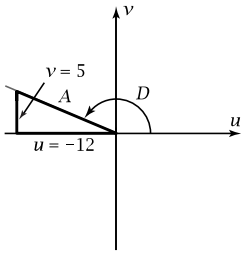


6. Show that the angle D in Problem 3 is a value of $\arctan \frac{4}{3}$, as shown in the figure in Problem 5.

7. Express as a cosine with a phase displacement:

   $y = -12 \cos \theta + 5 \sin \theta$

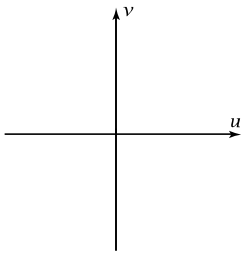
   Use the next uv-diagram to find the amplitude A and the phase displacement D. Show that D is a value of $\arctan \frac{5}{-12}$ but not the value of $\tan^{-1} \frac{5}{-12}$ that your calculator gives you.



   $y =$ _____

8. Express as a cosine with a phase displacement:

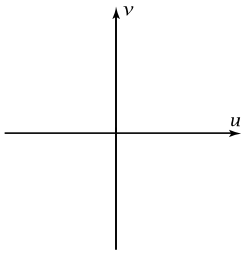
   $y = -6 \cos \theta - 11 \sin \theta$



   $y =$ _____

9. Express as a cosine with a phase displacement:

   $y = 9 \cos \theta - 7 \sin \theta$



   $y =$ _____

10. What did you learn as a result of doing this Exploration that you did not know before?