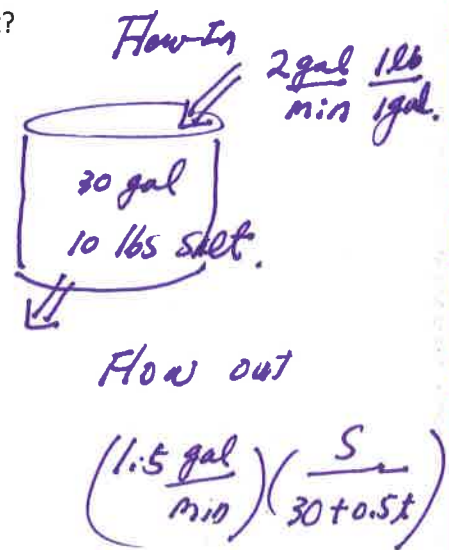Name: _____ **key** _____ Period: ____

A tank contains 10 lb of salt dissolved in 30 gal of water. Suppose 2 gal of brine containing 1 lb of dissolved salt per gallon runs into the tank every minutes and that the mixture (kept uniform by stirring) runs out of the tank at the rate of 1.5 gal/ min. How long does it take (to nearest second) for the tank to contain 15 lb of salt?

Flow In

$\frac{2 \text{ gal}}{\min} \frac{1 \text{ lb}}{1 \text{ gal}}$

30 gal
10 lbs salt.

Flow out

$\left(1.5 \frac{\text{gal}}{\min}\right)\left(\frac{S}{30+0.5t}\right)$

DE:

$$\frac{ds}{dt} = 2 - \frac{1.5\,(S_{(t)})}{30+0.5t}$$

$$\frac{ds}{dt} = 2 - \frac{3\,S_{(t)}}{60+t}$$

$$\Rightarrow \frac{ds}{dt} + \frac{3}{60+t}\,S_{(t)} = 2 \qquad \Rightarrow P = \frac{3}{60+t}$$

$$\Rightarrow (60+t)^3 \cdot S' + 3(60+t)^2 S = 2(60+t)^3 \qquad I_{(t)} = e^{\int \frac{3}{60+t}dt} = e^{3\ln(60+t)}$$

$$= (60+t)^3$$

$$\Rightarrow \frac{d}{dt}\left[S(60+t)^3\right] = 2(60+t)^3$$

$$\Rightarrow \int d\left[S(60+t)^3\right] = \int 2(60+t)^3 dt$$

$$S = \frac{1}{(60+t)^3}\left[\frac{2}{4}(60+t)^4 + C\right] \qquad S_{(0)} = 10 \text{ lbs}$$

$$C = (-20)(60)^3$$

$$\boxed{S = \frac{1}{2}(60+t) - (20)(60)^3(60+t)^{-3}} \qquad \boxed{t \approx 3.59 \; (3\min 36 \sec)}$$

$$15 = \frac{1}{2}(60+t) - 20(60)^3(60+t)^{-3} \Rightarrow \text{Solve by GDG}$$