WS # 3 ①

IB Math HL1 Application for Linear Differential Equations: **Name:** solutions

a. A tank initially contains 10 kg of dissolved salt in 300 litres of water. The solution runs out at the rate of 3 litres/min. Fresh water is added into the tank at the same rate.

Let x kg of salt be present in the tank at any time t minutes.

i. Find the concentration of salt in the tank at any time t minutes.
ii. Find the rate, in kg/min, at which salt runs out of the tank.
iii. Set up the differential equation for the amount of salt in the tank at any time t minutes.
iv. Solve this d.e. and find how long it takes for the concentration of salt in the tank to reach 40% of its initial concentration.

b. A salt solution of 0.2 kg/litre is now entering the tank and the solution runs in and out at the same rate as before.

i. Set up the differential equation for this situation.
ii. Assuming the same initial conditions as in part a., how much salt will there be in the tank after 2 hours?

c. Assume that for the situation described in part b., the rate at which the salt/water solution runs in is 2 litres/min but still runs out at 3 litres/min.

a. (i) $\left(\frac{x}{300}\ \frac{kg}{l}\right)$ (ii) $\left(\frac{3l}{min}\right)\left(\frac{x}{300}\ \frac{kg}{l}\right) = \left(\frac{x}{100}\right)\frac{kg}{min}$.

(iii) $\frac{dx}{dt} = -\frac{x}{100}\ \frac{kg}{min}$ (iv) $\frac{dx}{x} = \left(\frac{-1}{100}\right)dt \Rightarrow \ln x = \frac{-1}{100}t + C$

$\boxed{x = 10 \cdot e^{-\frac{1}{100}t}}$

$t \geq 0$

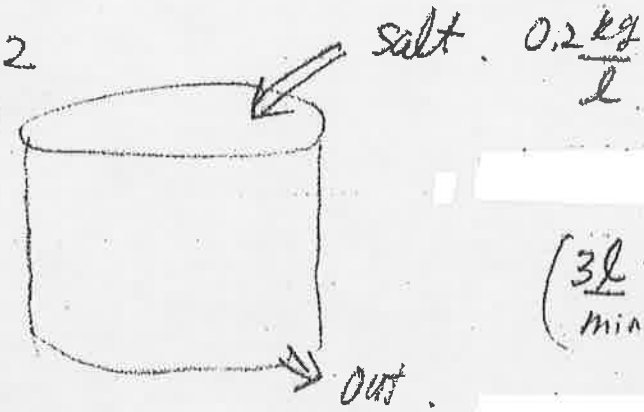
40% of 10kg $\Rightarrow 4 = 10 \cdot e^{-\frac{1}{100}t}$

$t = 91.63$ min

b. and c are attached.

#2

(b)



∘ amount of salt flow In

$$\left(\frac{3\,l}{min}\right)\left(0.2\frac{kg}{l}\right) = \boxed{0.6\ \frac{kg}{min}}$$

∘ amount salt flow out.

$$\left(3\frac{l}{min}\right)\left(\frac{x}{300}\right) = \frac{x}{100}\ \frac{kg}{min}$$

(i) $\boxed{\frac{dx}{dt} = 0.6 - \frac{x}{100} \quad (x(0)=10)}$

$$\Rightarrow \frac{dx}{dt} = \frac{60-x}{100}$$

(ii) $\Rightarrow \frac{dx}{60-x} = \frac{dt}{100} \Rightarrow -\ln|60-x| = \frac{t}{100} + c \qquad x=10\ \ t=0$

$$c = -\ln 50$$

$$\Rightarrow \frac{t}{100} = \ln\left(\frac{50}{60-x}\right) \Rightarrow \boxed{x = 60 - 50e^{-\frac{1}{100}t}}$$ ✓

$$\Rightarrow \frac{50}{60-x} = e^{\frac{t}{100}} \Rightarrow 50 = e^{\frac{t}{100}}(60-x)$$

When $t = 120$ minutes $\quad x = 60 - 50\cdot e^{-\frac{1}{100}(120)} \approx \boxed{44.94\ kg}$ ✓

(c) The Rate of amount salt flow-in

$$\left(2\frac{l}{min}\right)\left(0.2\frac{kg}{l}\right) = 0.4\frac{kg}{l}$$

The Rate of amount Salt flow out

$$\left(3\frac{l}{min}\right)\left(\frac{x}{300+(3-2)t}\right) = \frac{3x}{300+t}$$

$$\Rightarrow \boxed{\frac{dx}{dt} = 0.4 - \frac{3x}{300+t} \quad (x(0)=10)}$$