IB Math 3: Maclaurin Polynomial **Name:** Key **Period:** ______

Group Activity

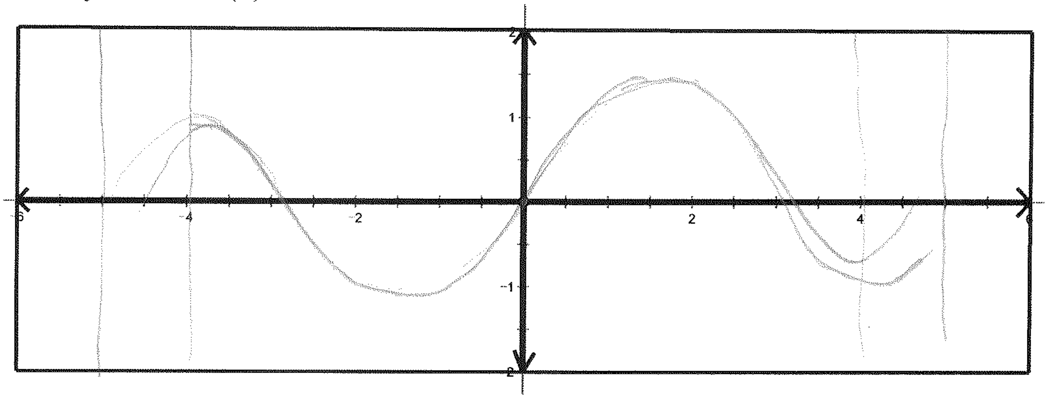
Part I: *Discovering another function similar to a polynomial within a limited x interval.*

1. Graph the function $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$ in your Graphing Utility using the window: $x:[\cancel{-6,6}]$ $-5,5$ and $y:[-2,2]$.

2. Describe the graph of f(x). What other function does this remind you of? Name this other function, g(x).

   $g(x) = \sin x$

3. Graph f(x) and g(x) simultaneously at the following window and determine the interval of x for which g(x) is approximately same as f(x).



g(x) is approximately equal to f(x) in the x interval [-4 **,** 4 **]**

OR[3.5, 3.5]

Part II: *Developing a polynomial function.*

1. Using the following clues, find a third degree polynomial function; $f(x) = ax^3 + bx^2 + cx + d$.

$$f(0) = -1, \quad f'(0) = 5, \quad f''(0) = -12, \quad f'''(0) = 18$$

Work:

$d = -1$

$f'(x) = 3ax^2 + 2bx + c$ $\quad c = 5$

$f''(x) = 6ax + 2b$ $\quad b = -6$

$f'''(x) = 6a$ $\quad 6a = 18$ $\quad a = 3$

$f(x) =$ $3x^3 - 6x^2 + 5x - 1$

Part III: *Estimating* $h(0.5)$ *where h(x) is unknown function.*

1. Your teacher is thinking a function and wants you and your partner to calculate the value of the function at $x = 0.5$ with the given clues. The function is not polynomial, but has the following properties as the follows:

$$f(0)=2,\quad f'(0)=-2,\quad f''(0)=4,\quad f'''(0)=-12,\quad f^{iv}(0)=48$$

Now, pretend the function is a 4th degrees of polynomial, h(x), and find the polynomial that satisfies the above conditions.

$y = ax^4 + bx^3 + cx^2 + dx + e$

$e = 2$

$y' = 4ax^3 + 3bx^2 + 2cx + c$

$d = -2$

$y'' = 12ax^2 + 6bx + 2c$

$2c = 4 \Rightarrow c = 2$

$y''' = 24ax + 6b$

$6b = -12 \quad b = 2$

$y^4 = 24a$

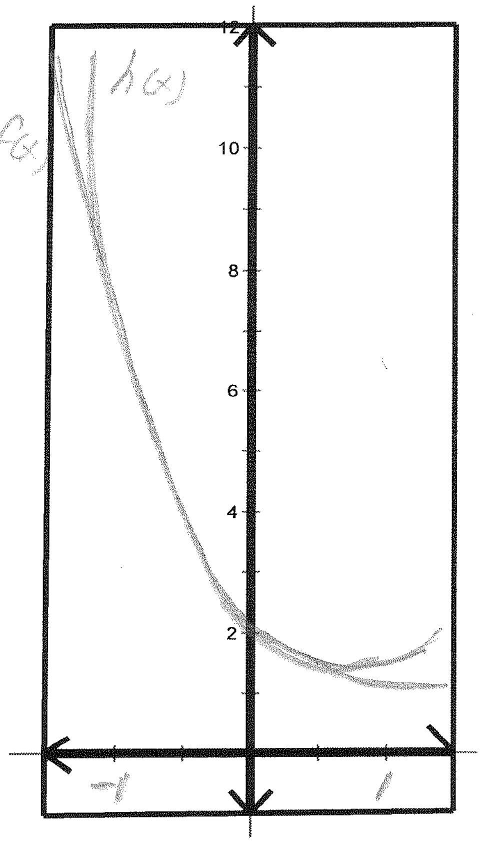
$24a = 48 \quad a = 2$

$h(x) = 2x^4 - 2x^3 + 2x^2 - 2x + 2$

$h(0.5) = 1.375$

2. Then find $h(0.5)$. How close do you think $h(0.5)$ is to the actual function value at $x = 0.5$.

$h(0.5) = 1.375$



3. Check with your teacher and graph both the function and the polynomial at the provided window.

$h(x) = 2x^4 - 2x^3 + 2x^2 - 2x + 2$

$f(x) = \dfrac{2}{x+1} \qquad f(0.5) = 1.333$

Part IV: Given the function: $f(x) = e^{2x}$ +1

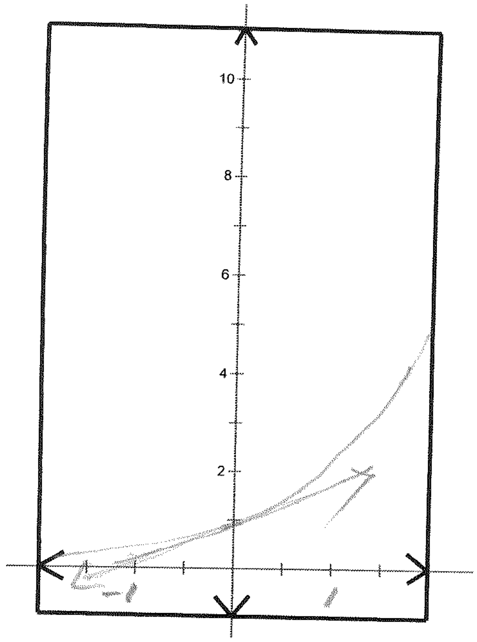
1. What is the equation of the line tangent to $f(x)$ at $x = 0$? Graph the tangent line and $f(x)$ simultaneously.

$f'(x) = 2e^{2x}$

$y = 2x$ $x = 0$

$y = 2$

$y = 2x + 2$



2. Find the second degree polynomial $p(x) = ax^2 + bx + c$ Which will imitate $f(x)$ around $x = 0$. Graph p(x) and $f(x)$ simultaneously.

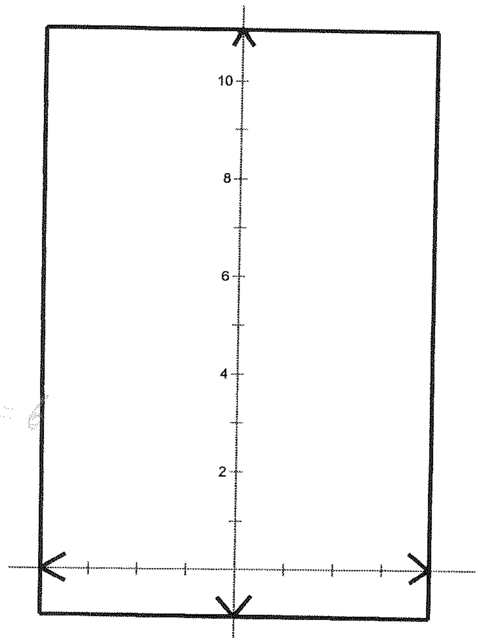
$f'(x) = 2e^{2x}$ $f'(0) = 2$ $c = 2$

$f''(x) = 4e^{2x}$ $f''(0) = 4$ $b = 4$

$f'''(x) = 6e^{2x}$ $f'''(0) = 6$ $P'''(x) = 2a = 6$

$a = 3$

$P(x) = 3x^2 + 4x + 2$



3. Compare the errors of the linear and quadratic polynomials as they approximate $f(-0.5)$ and $f(0.5)$. Do you think you might have a better approximation $f(0.5)$ if you find a higher degree of polynomial imitating $f(x)$?

Part V: Maclaurin Polynomial

1. Investigations of Part II and III both involved finding a polynomial given only derivatives values. Now derive a general polynomial to imitate a function $f(x)$ about the point $(0, f(0))$. A polynomial of this type is called a MacLaurin Polynomial.

a) The line tangent to $f(x)$ at $x = 0$ is the first degree polynomial at $x = 0$; $p_1(x) = f(0) + f'(0)x$

b) Let's call the quadratic polynomial; $p_2(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!}$

c) Guess the 3rd degree polynomial $p_3(x)$ and write out;

2. The formula of Maclaurin Polynomial for $p_n(x)$, where $n \in Z^+$ is:

$$\boxed{p_n(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} \ldots\ldots = \sum_{k=0}^{\infty} \frac{f^k(0)x^k}{k!}}$$

3. Practice Problems

a. Write the Maclaurin series for $\sin x$, And then the series of $\sin x$ using $\sum$ notation.

$f(x) = \sin x.$

$f(0) = \sin 0 = 0$

$f'(0) = \cos 0 = 1$

$f''(0) = -\sin 0 = 0$

$f'''(0) = -\cos 0 = -1$

$f^4(0) = +\sin 0 = 0$

$f^5(0) = \cos 0 = 1$

$$\Rightarrow p(x) = 0 + x - \frac{0x^2}{2!} + \frac{(-1)x^3}{3!} + \frac{(0)x^4}{4!} + \frac{(1)x^5}{5!} \ldots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$$

b. Write the Maclaurin series for $\arctan x$, And then the series of arctan x using $\sum$ notation.

Answer attached

#3.

b $f(x) = \tan^{-1} x$

$f(0) = 0$ $\qquad$ $f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$

$f'(0) = \frac{1}{1+0^2} = 1$

$f''(x) = (-1)(1+x^2)^{-2} \cdot 2x \Rightarrow f''(0) = 0$

$f'''(x) = 2(1+x^2)^{-3} \cdot (2x)^{4} + (-1)(1+x^2)^{-2} \cdot 2 \Rightarrow f'''(0) = -2$

$f^{4}(x) = -6(1+x^2)^{-4}(2x)(2x)^{4} + 2(1+x^2)^{-3}(2) 4x^3 + 2(2)(1+x^2)^{-3}(2x)$

$f^{4}(0) = 0$

$$\Rightarrow \quad p(x) = 0 + 1 \cdot x + \frac{0 \cdot x^2}{2!} - \frac{2 \cdot x^3}{3!} + \frac{0 \cdot x^4}{4!} + \cdot \frac{\quad}{5!}$$

$$= x - \frac{x^3}{3} + 0 - \frac{x^5}{5} \cdots$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{2n+1}$$