IB Math HL1: Mixed Review for more Practice

Name: Solution. Period: 0

Find the derivatives of the following functions.

1. $f(x) = (2-3x)^2 + 2x\sqrt{x}$

$= (2-3x)^2 + 2x^{3/2}$

$\frac{df}{dx} = (2)(-3)(2-3x) + 2\left(\frac{3}{2}\right)x^{1/2}$

$\boxed{\frac{df}{dx} = -6(2-3x) + 3\sqrt{x}}$

2. $f(x) = \dfrac{x^3 - \sqrt{x}}{x} = x^2 - x^{-\frac{1}{2}}$

$\frac{df}{dx} = 2x + \frac{1}{2}x^{-\frac{3}{2}} = \boxed{2x + \frac{1}{2\sqrt{x}\,x}}$

OR $= \boxed{2x + \frac{\sqrt{x}}{2x^2}}$

3. $f(x) = \sqrt[3]{x}(5x^2-2)^3$

$\frac{df}{dx} = \frac{1}{3}x^{-\frac{2}{3}}(5x^2-2)^3 + \sqrt[3]{x}\cdot 3\cdot(10x)(5x^2-2)^2$

$= \boxed{\frac{(5x^2-2)^3}{3\sqrt[3]{x^2}} + 30x\sqrt[3]{x}(5x^2-2)^2}$

4. $f(x) = \dfrac{x^3}{5x^2-8}$

$\frac{df}{dx} = \frac{3x^2(5x^2-8) - x^3(10x)}{(5x^2-8)^2}$

$\frac{df}{dx} = \frac{15x^4 - 24x^2 - 10x^4}{(5x^2-8)^2} = \boxed{\frac{5x^4-24x^2}{(5x^2-8)^2}}$

5. $f(x) = \left(\dfrac{x-6}{x+7}\right)^3$

$\frac{df}{dx} = 3\left(\frac{x-6}{x+7}\right)^2\left[\frac{(x+7)-(x-6)}{(x+7)^2}\right]$

$= \frac{3(x-6)^2(13)}{(x+7)^4} = \boxed{\frac{39(x-6)^2}{(x+7)^4}}$

6. Find the gradient of the tangent to the curve of $f(x) = \dfrac{5x^3}{(2x+1)^2}$ at x=1

$\frac{df}{dx} = \frac{15x^2(2x+1)^2 - (5x^3)(2)(2)(2x+1)}{(2x+1)^4}$

$\left.\frac{df}{dx}\right|_{x=1} = \frac{(15)(3)^2 - (5)(4)(3)}{3^4} = \boxed{\frac{25}{27}}$: Gradient at $x=1$

7. Find the coordinate of x where the gradient of the tangent is 0 for $f(x) = \dfrac{x}{\sqrt{1-3x}}$.

$\frac{df}{dx} = \frac{\sqrt{1-3x} - x\left(\frac{1}{2}\right)(-3)(1-3x)^{-\frac{1}{2}}}{1-3x}$.

$= \frac{\left(\sqrt{1-3x} + \frac{3x}{2\sqrt{1-3x}}\right)\cdot[2\sqrt{1-3x}]}{(1-3x)\cdot[2\sqrt{1-3x}]} = \frac{2(1-3x)+3x}{2\sqrt{1-3x}(1-3x)} = \frac{2-3x}{2(\sqrt{1-3x})(1-3x)}$

$\frac{df}{dx} = 0 = \frac{2-3x}{2\sqrt{1-3x}(1-3x)} \Rightarrow 2-3x = 0 \quad \boxed{x = \frac{2}{3}}$