

key.

IB Math HL1: More Equation(s) of the tangent line

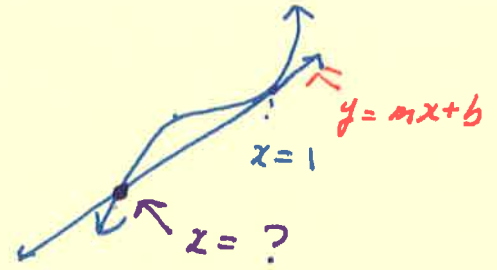
Example 1) Find the x-coordinates of the point(s) where the tangent to $f(x) = x^3 + x + 2$ at $x=1$ meets the curve again.

$$\frac{dy}{dx} = 3x^2 + 1$$

$$\frac{dy}{dx} \Big|_{x=1} = 3(1)^2 + 1 = 4$$

$$x=1 \quad f(1) = 1 + 1 + 2 = 4 = y.$$

$$\Rightarrow y - 4 = 4(x - 1).$$



$f(x) = y \Rightarrow$ solve for x

$$y = 4x - 4 + 4 = 4x.$$

$$\Rightarrow x^3 + x + 2 = 4x.$$

$$\textcircled{1} x^3 - 3x + 2 = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & -2 & 0 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$$x^3 - 3x + 2 = (x-1)(x^2 + x - 2)$$

$$= (x-1)(x+2)(x-1)$$

$$x = 1, x = -2$$

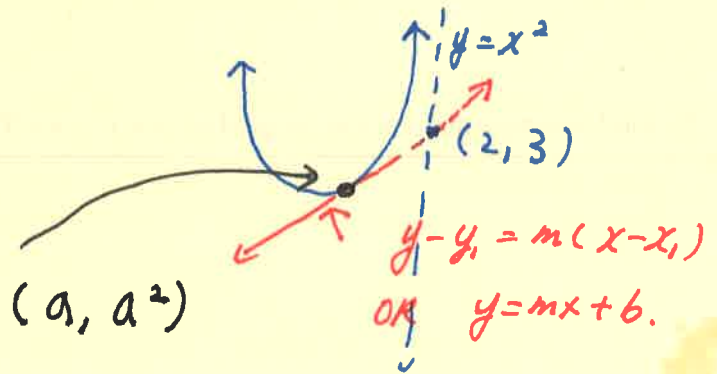
Example 2) Find the equations of tangents to $y = x^2$ from the external point $(2, 3)$

$$\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} \Big|_{x=a} = 2a.$$

$$\Rightarrow y - a^2 = 2a(x - a)$$

Tangent line.



$$(2, 3) \quad \textcircled{1} a = 1 \quad y - a^2 = 2a(x - a)$$

$$y - 1 = 2(x - 1) \text{ OR } y = 2x - 1$$

$$\textcircled{2} a = 3$$

$$y - 9 = 6(x - 3) \text{ OR } y = 6x - 9$$

$$3 - a^2 = 2a(2 - a)$$

$$3 - a^2 = 4a - 2a^2$$

$$a^2 - 4a + 3 = 0$$

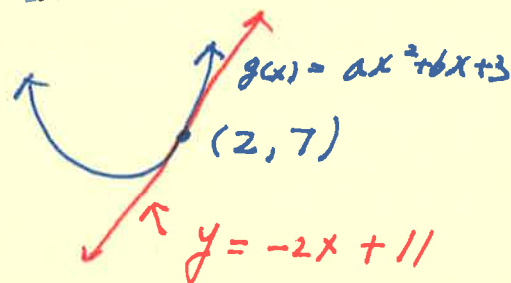
$$(a - 3)(a - 1) = 0$$

$$a = 3, a = 1$$

Example 3) $y = -2x + 11$ is the equation of a tangent line to the graph of $g(x) = ax^2 + bx + 3$ at $(2, 7)$. Find the value of a and b .

$$\frac{dy}{dx} = 2ax + b.$$

$$\frac{dy}{dx} \Big|_{x=2} = \boxed{4a + b}$$



$$\textcircled{1} \quad \boxed{4a + b = -2} \quad \textcircled{1}$$

$$g(2) = 7 = a(2)^2 + b(2) + 3 = 4a + 2b = 7 - 3$$

$$4a + b = -2$$

$$2a + b = 2$$

$$2a = -4$$

$$a = -2$$

$$b = 6$$

$$4a + 2b = 4$$

$$\boxed{2a + b = 2} \quad \textcircled{2}$$

Practice)

1) Find the equation of tangents to $y = x^3$ from the external point $(-2, 0)$.

$$\frac{dy}{dx} = 3x^2$$

(contact point (a, a^3))

\Rightarrow Equation of tangent at (a, a^3)

$$y = 3a^2(x - a) + a^3$$

\rightarrow This line contains $(-2, 0)$

$$0 = 3a^2(-2 - a) + a^3$$

$$0 = -6a^2 - 3a^3 + a^3$$

$$2a^3 + 6a^2 = 0$$

$$2a^2(a + 3) = 0$$

$$a = 0 \quad a = -3.$$

Equations of tangent

$$y = 0$$

$$y = 27x + 54$$

2) Find two equations of the tangent to $y = x^2 - x + 9$ from the external point $(0, 0)$

Find the tangent at $x = a$. $y = a^2 - a + 9$

$$\frac{dy}{dx} = 2x - 1$$

$$\frac{dy}{dx} \Big|_{x=a} = 2a - 1$$

$\Rightarrow y = (2a - 1)(x - a) + a^2 - a + 9 \in$ contains $(0, 0)$

$$\Rightarrow 0 = (2a - 1)(-a) + a^2 - a + 9$$

$$0 = -2a^2 + a + a^2 - a + 9$$

$$a^2 = 9 \quad a = \pm 3$$

Equations of tangent

$$y = 5x \quad (x = 3)$$

$$y = -7x \quad (x = -3)$$