IB Math 3:

Name: Sample Work **Period:** ______

Estimation of Alternating Series and Power Series review

Practice:

Find the radius of convergence and the interval of convergence for the following series:

1) $\sum_{n=1}^{\infty}\left(\frac{(x-3)^n}{n}\right)$

Ratio test:

$$\lim_{n\to\infty}\left(\frac{(x-3)^{n+1}}{n+1}\right)\left(\frac{n}{(x-3)^n}\right)$$

$$=\lim_{n\to\infty}\left(\frac{\cancel{(x-3)^n}(x-3)}{n+1}\right)\left(\frac{n}{\cancel{(x-3)^n}}\right)$$

$$=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)(x-3) = \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)(x-3)$$

$$= x-3$$

$|x-3|<1 \Rightarrow -1<x-3<1$

$\boxed{R=1}$ $\boxed{2<x<4}$

$x=2$: $\sum \frac{(2-3)^n}{n}$

$=\sum \frac{(-1)^n}{n} \Rightarrow$ Alt. Series test $\lim_{n\to\infty}\frac{1}{n}=0$

$\left(\frac{1}{n}\right)' = \frac{-1}{n^2} < 0$ decreasing.

Conditionally converges.

$x=4$: $\sum \frac{(4-3)^n}{n} = \sum \frac{1}{n}$

Diverges by p series $p=1$.

$\therefore \boxed{2 \le x < 4}$

$[2, 4[$

2) $\sum_{n=0}^{\infty}\left(\frac{(-3)^n x^n}{\sqrt{n+1}}\right)$

Attached.

3) $\sum_{n=1}^{\infty}\left(\frac{(-1)^n x^{2n-1}}{(2n-1)!}\right)$

4) $\sum_{n=1}^{\infty}\left(\frac{(-1)^n (2x+3)^n}{n \ln n}\right)$

#2

Ratio test:

$$\lim_{n\to\infty}\left(\frac{(-3)^{n+1}(x)^{n+1}}{\sqrt{n+2}}\right)\left(\frac{\sqrt{n+1}}{(-3)^n(x)^n}\right) = \lim_{n\to\infty}\left(\frac{(-3)^n\cdot(-3)\cancel{x^n}\cdot x}{\sqrt{n+2}}\right)\left(\frac{\sqrt{n+1}}{\cancel{(-3)^n}\cancel{(x)^n}}\right)$$

$$= \lim_{n\to\infty}(-3)\left(\sqrt{\frac{n+1}{n+2}}\right) \begin{matrix}\div\sqrt{n}\\ \div\sqrt{n}\end{matrix}$$

$$= \lim_{n\to\infty}(-3x)\sqrt{\frac{1+\frac{1}{n}}{1+\frac{2}{n}}} = -3x.$$

$$\Rightarrow \quad |3x|<1$$

$$-1<3x<1$$

$$-\frac{1}{3}<x<\frac{1}{3} \qquad R=\frac{1}{3}.$$

② $x=\frac{1}{3}$. $\sum \frac{(-3)^n(\frac{1}{3})^n}{\sqrt{n+1}} = \sum \frac{(-1)^n}{\sqrt{n+1}}$

$\cancel{\sum \frac{1}{\sqrt{n+1}}}$ $\quad \cancel{\sum}$ Alt Series tes.

$$\lim_{n\to\infty}\frac{1}{\sqrt{n+1}}=0$$

$\frac{1}{\sqrt{n+1}}$ is decreasing.

$\Rightarrow$ Conditionally Converges.

① $x=\frac{-1}{3}$ $\quad \sum \frac{(-3)^n(-\frac{1}{3})^n}{\sqrt{n+1}} = \sum \frac{1^n}{\sqrt{n+1}}$

$\therefore \boxed{-\frac{1}{3}<x\leq\frac{1}{3}}$ $\qquad \sum\frac{1}{\sqrt{n+1}} < \sum\frac{1}{\sqrt{n}}$

Limit comparison $\quad \lim_{n\to\infty}\frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}}=1$

$(-\frac{1}{3},\frac{1}{3}]$

#3.

Ratio test:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{(-1)^{n+1}\cdot x^{2(n+1)-1}}{(2(n+1)-1)!}\right]\left[\frac{(2n-1)!}{(-1)^n\cdot x^{2n-1}}\right]$$

$$= \lim_{n\to\infty} \left(\frac{\cancel{(-1)^n}(-1)\, x^{\cancel{2n-1}}\cdot x^2}{\underbrace{(2n+2-1)!}_{(2n+1)!}}\right)\left(\frac{(2n-1)!}{\cancel{(-1)^n}\cdot x^{\cancel{2n-1}}}\right)$$

$$= \lim_{n\to\infty} \frac{(-1)x^2\cdot \cancel{(2n-1)!}}{\cancel{(2n-1)!}(2n)(2n+1)} = \lim_{n\to\infty} \frac{x^2(-1)}{(2n)(2n+1)} = \frac{x^2}{\infty} = 0 < 1.$$

$R = \infty$

Interval : $(-\infty, \infty)$.

#4. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n (2x+3)^n}{n \ln n}$.

Ratio test.

$$\lim_{n\to\infty} \left[\frac{(-1)^{n+1}(2x+3)^{n+1}}{(n+1)(\ln(n+1)}\right]\left[\frac{n \ln n}{(-1)^n(2x+3)^n}\right]$$

$$= \lim_{n\to\infty} \left(\frac{\cancel{(-1)^n}\cdot(-1)\,\cancel{(2x+3)^n}(2x+3)}{\cancel{(-1)^n}\,\cancel{(2x+3)^n}}\right)\left(\frac{n \cdot \ln n}{(n+1)\cdot \ln(n+1)}\right)$$

$$= \lim_{n\to\infty} (-1)(2x+3) \cdot \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{1} \cdot \lim_{n\to\infty}\left(\frac{\ln n}{\ln(n+1)}\right)^{1}$$

$$= (-1)(2x+3)$$

$$\Rightarrow |2x+3| < 1 \Rightarrow -1 < 2x+3 < 1 \Rightarrow -4 < 2x < -2$$

$$\Rightarrow -2 < x < -1 \Rightarrow R = \frac{1}{2}. \Rightarrow \therefore \text{Interval. } \boxed{(-2, -1]}$$

End points.

$x = -2. \Rightarrow \displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n(-1)^n}{n \ln n}.$

$= \displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}.$

Int. test. $\displaystyle\lim_{a\to\infty} \int_2^a \frac{1}{x \cdot \ln x}\,dx.$ $u = \ln x$, $du = \frac{1}{x}\,dx$

$= \displaystyle\lim_{a\to\infty} \int_{\ln 2}^{a} \frac{1}{u}\,du = \lim_{a\to\infty} [\ln a - \ln(\ln 2)] = \infty$

diverges.

$x = -1 \Rightarrow \displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n (1)^n}{n \cdot \ln n}.$

Alt. Series test.

$\displaystyle\lim_{n\to\infty} \frac{1}{n \cdot \ln n} = 0$

$\frac{1}{n \ln n} \Rightarrow$ decreasing

$\frac{1}{2\ln 2} > \frac{1}{3\ln 3} > \frac{1}{4\ln 4} \cdots$

converges.

#5. $\sum_{n=1}^{\infty} \left(\frac{1}{5n^2}\right)$

$$\Rightarrow \sum_{n=1}^{8} \frac{1}{5n^2} = \frac{1}{1^2\cdot 5} + \frac{1}{2^2\cdot 5} + \frac{1}{3^2\cdot 5} + \frac{1}{4^2\cdot 5} + \frac{1}{5^2\cdot 5} + \frac{1}{6^2\cdot 5} + \frac{1}{7^2\cdot 5} + \frac{1}{8^2\cdot 5}. \approx 0.259832$$

$$0.259832 - \frac{1}{9^2\cdot 5} \le S \le 0.259832 + \frac{1}{9^2\cdot 5}$$

$$0.257363 \le S \le 0.262301$$

Remainder

#6. $$\frac{1}{n^4} \le 0.0000005$$

1 2 3 4 5 6.

$$n^4 \ge \frac{1}{0.0000005}$$

$$n \ge \sqrt[4]{\frac{1}{0.0000005}} \approx 37.6$$

$$n = 38$$