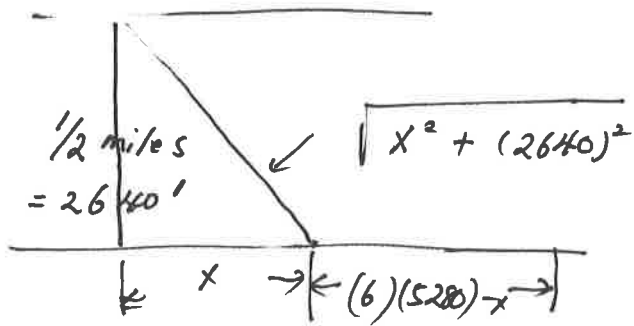


Solutions (Optimization W.S)

①

#1.

1)



2)

$$C(x) = 8\sqrt{x^2 + (2640)^2} + 6[(6 \cdot 5280) - x]$$

5280 feet = 1 mile

$$3) \frac{dC}{dx} = \frac{(8)(\frac{1}{2})(2x)}{\sqrt{x^2 + (2640)^2}} - 6 = 0$$

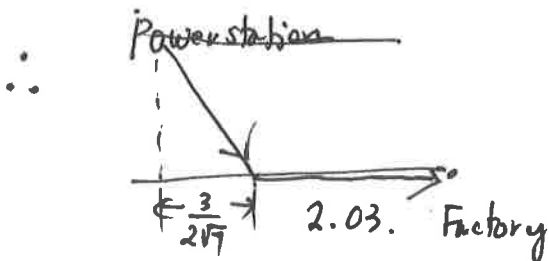
$$\Rightarrow \frac{8x}{\sqrt{x^2 + (2640)^2}} = 6 \Rightarrow \left(\frac{4}{3}x\right)^2 = \left(8\sqrt{x^2 + (2640)^2}\right)^2$$

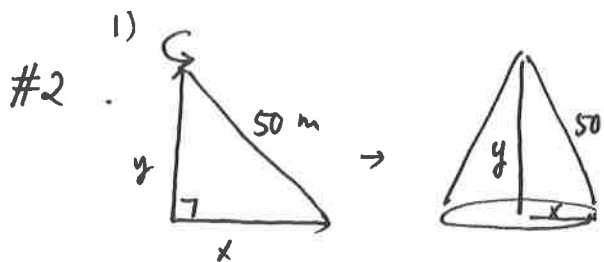
$$\Rightarrow 16x^2 = 9(x^2 + (2640)^2)$$

$$\Rightarrow 7x^2 = (9)(2640)^2$$

$$x = \sqrt{\frac{(9)(2640)^2}{7}} = \left(\frac{(3)(2640)}{\sqrt{7}}\right) \text{ feet} \left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)$$

$$= \frac{3}{2\sqrt{7}} \text{ miles.}$$





2)

$$V = \frac{1}{3}\pi x^2 y$$

$$x^2 + y^2 = 50^2$$

$$x^2 = 50^2 - y^2$$

$$V = \frac{1}{3}\pi (50^2 - y^2) \cdot y$$

$$= \frac{1}{3}\pi (50^2 y - y^3)$$

3)

$$\frac{dV}{dy} = \frac{1}{3}\pi [50^2 - 3y^2] = 0$$

$$3y^2 = 50^2 \Rightarrow y = \frac{50}{\sqrt{3}} \text{ m}$$

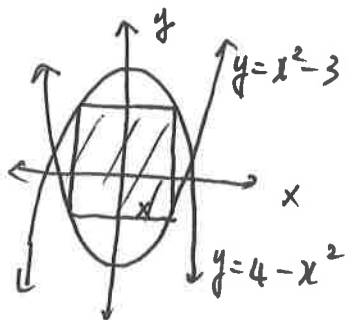
4) Check.

$$\frac{d^2V}{dy^2} = \frac{1}{3}\pi [-6y]$$

$$\frac{d^2V}{dy^2} \Big|_{y=\frac{50}{\sqrt{3}}} = \frac{1}{3}\pi \left[-6 \left(\frac{50}{\sqrt{3}}\right)\right] < 0 \quad \curvearrowright \text{ Max Value.}$$

$$\therefore V = \frac{1}{3}\pi \left[50^2 - \left(\frac{50}{\sqrt{3}}\right)^2\right] \frac{50}{\sqrt{3}}$$

#3. 1)



2)

$$A = 2x \cdot y$$

$$y = (4 - x^2) - (x^2 - 3)$$

$$= -2x^2 + 7$$

$$A = 2x(-2x^2 + 7) = -4x^3 + 14x$$

3)

$$\frac{dA}{dx} = 0 \quad \text{solve for } x.$$

4) Check.

$$\frac{d^2A}{dx^2} = -24x$$

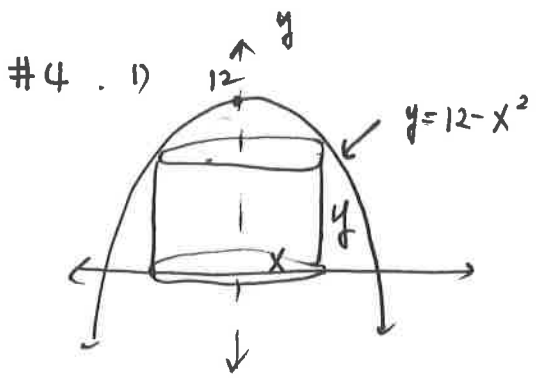
$$\frac{dA}{dx} = -12x^2 + 14 = 0$$

$$x = \sqrt{\frac{7}{6}}$$

$$\frac{d^2A}{dx^2} \Big|_{x=\sqrt{\frac{7}{6}}} = -24\left(\sqrt{\frac{7}{6}}\right) < 0$$

$$\therefore A = -4\left(\sqrt{\frac{7}{6}}\right)^3 + 14\left(\sqrt{\frac{7}{6}}\right)$$

\curvearrowright
Max Value



2)

$$V = \pi r^2 h$$

$$= \pi x^2 \cdot y$$

$$y = 12 - x^2$$

$$V = \pi x^2 (12 - x^2) = 12\pi x^2 - \pi x^4$$

3) $\frac{dV}{dx} = 0$ solve for x

$$\frac{dV}{dx} = \pi (24x - 4x^3) = 0 = 4\pi (6 - x^2) = 0$$

~~x=0~~, $x = \pm\sqrt{6}$.

4) Check $\frac{d^2V}{dx^2} = \pi (24 - 12x^2)$

$$\frac{d^2V}{dx^2} = \pi (24 - 12(\sqrt{6})^2) < 0 \quad \checkmark \quad \text{Max Value}$$

$\therefore V = \pi (\sqrt{6})^2 (12 - \sqrt{6}^2) = \pi \cdot 6 \cdot 6 = \boxed{36\pi \text{ unit}^3}$