Notes.

Derivatives of Trig functions

$$\boxed{\frac{d\sin x}{dx} = \cos x} \qquad \boxed{\frac{d\cos x}{dx} = -\sin x}$$

$$\boxed{\frac{d\tan x}{dx} = \sec^2 x} \qquad \boxed{\frac{d\sec x}{dx} = \sec x \cdot \tan x}$$

⇒ proof: $\boxed{\frac{d\sin x}{dx} = \cos x}$

Notes: Identity: $\boxed{\sin(a+b) = \sin a\cos b + \cos a \cdot \sin b}$

$$\boxed{\lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}} = \lim_{\Delta x \to 0} \frac{\sin x\cos\Delta x + \cos x\sin\Delta x - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin x[\cos\Delta x - 1] + \cos x \cdot \sin\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \underbrace{\frac{\sin x[\cos\Delta x - 1]}{\Delta x}}_{0} + \lim_{\Delta x \to 0} \cos x \cdot \underbrace{\frac{\sin\Delta x}{\Delta x}}_{1}$$

$$= \boxed{\cos x}$$

⇒ proof: $\boxed{\frac{d\cos x}{dx} = -\sin x}$.

Using Identity and the Chain Rule

$$y = \cos x = \sin\left(\frac{\pi}{2} - x\right) = \sin u \quad \text{where } u = \frac{\pi}{2} - x$$

$$\frac{d\sin u}{dx} = \frac{d\sin u}{du} \cdot \frac{du}{dx} = \cos u \cdot (-1) = -\cos u$$

$$\Rightarrow -\cos u = -\cos\left(\frac{\pi}{2} - x\right) = \boxed{-\sin x}$$

→ proof for $\boxed{\frac{d \tan x}{dx} = \sec^2 x}$ Using the Quotient Rule.

$$\frac{d \tan x}{dx} = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{(\sin x)'(\cos x) - (\sin x)(\cos x)'}{(\cos x)^2}$$

$$= (\cos x)(\cos x) - (\sin x)(-\sin x)$$

$$= \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

→ proof for $\boxed{\frac{d \sec x}{dx} = \sec x \cdot \tan x}$ Using the Quotient Rule

$$\frac{d \sec x}{dx} = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{(1)'\cos x - (1)(\cos x)'}{(\cos x)^2}$$

$$= \frac{0 - (-\sin x)}{(\cos x)^2} = \left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)$$

$$= \boxed{\tan x \cdot \sec x}$$

ex) Differentiate with respect to x.

a) $3x^2 \sin x$ Answer: $\boxed{6x \sin x + 3x^2 \cos x}$

b) $4 \tan(3x^3)$ Answer: $4 \sec^2(3x^3) \cdot 9x^2$

$$= \boxed{36x^2 \sec^2(3x^3)}$$

c) $e^{5x^2} \cdot \cos(2x+5)$ Answer: $\boxed{10x e^{5x^2} \cos(2x+5) - 2e^{5x^2} \sin(2x+5)}$