Work on Optimization

#1. $y = 4 - x^2$ $(0, 2)$ $(x, 4-x^2)$

$$d = \sqrt{x^2 + (2 - 4 + x^2)^2}$$

$$= \sqrt{x^2 + (x^2-2)^2} = \sqrt{x^2 + x^4 - 4x^2 + 4} = \sqrt{x^4 - 3x^2 + 4}$$

$$\Rightarrow d'(x) = \frac{1}{2}(x^4 - 3x^2 + 4)^{-\frac{1}{2}}(4x^3 - 6x) = 0$$

$$x = 0 \quad x = \pm\sqrt{\frac{6}{4}} = \pm\sqrt{\frac{3}{2}}$$

$$d''(x) = -\frac{1}{4}(x^4 - 3x^2 + 4)^{-1}(4x^3 - 6x) + \frac{1}{2}(x^4 - 3x^3 + 4)^{-\frac{1}{2}}(12x^2 - 6)$$

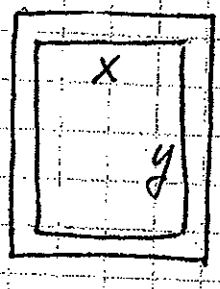
$$= \frac{-(4x^3 - 6x)}{4(x^4 - 3x^2 + 4)} + \frac{(12x^2 - 6)}{2\sqrt{x^4 - 3x^3 + 4}}$$

When $x = 0$ $d''(0) = \frac{-6}{2\sqrt{4}} < 0$ ↩ Max.

$x = \pm\sqrt{\frac{3}{2}}$ $f''(\pm\sqrt{\frac{3}{2}}) > 0$ ↪ min.

$$\left(\pm\sqrt{\frac{3}{2}}, 4 - \frac{3}{2}\right) \Rightarrow \left(\pm\sqrt{\frac{3}{2}}, \frac{5}{2}\right)$$

#2. $A = 24$



$$xy = 24$$

$$A_{paper} = (x+2)\left(\frac{24}{x} + 3\right)$$

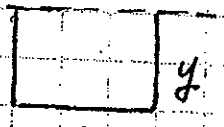
$$= 3x + \frac{48}{x} + 24 + 6 = 3x + \frac{48}{x} + 30$$

$$A'(x) = 3 - \frac{48}{x^2} = 0 \quad 3x^2 = 48 \quad x^2 = 16$$

$$x = \pm 4$$

$x = 4$.

#3.



$$A = x \cdot y = 180{,}000 \qquad y = \frac{180{,}000}{x}$$

$$P = 2y + x$$

$$P'(x) = \frac{-360000}{x^2} + 1$$

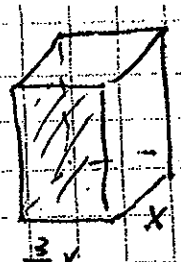
$$= \frac{360000}{x} + x$$

$$\Rightarrow x = \pm 600$$

$$x = 600$$

600 × 300.

#4



$\frac{3}{4}x^2 h = 900$

$h = \frac{1200}{x^2}$

$Cost = (4)\left(\frac{3}{4}x^2\right) + 6(x \cdot h) \cdot 2$

$+ 8\left(\frac{3}{4}x\right)(h) \cdot 2 + (3)\left(\frac{3}{4}x^2\right)$

$= 3x^2 + 12x\left(\frac{1200}{x^2}\right) + 9x\left(\frac{1200}{x^2}\right) + \frac{9}{4}x^2$

$= \frac{21}{4}x^2 + \frac{(12)(1200)}{x} + \frac{(9)(1200)}{x}$

$= \frac{21}{4}x^2 + \frac{25200}{x}$

$C'(x) = \frac{21}{2}x - \frac{25200}{x^2} = 0$

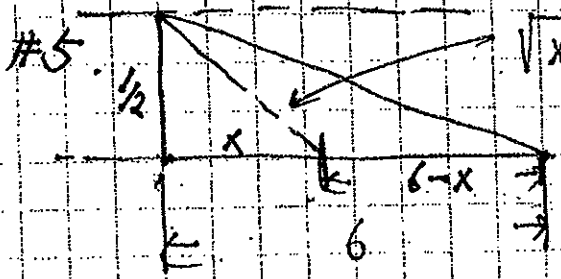
$\Rightarrow \frac{21x}{2} = \frac{25200}{x^2}$

$21x^3 = 2 \cdot (25200)$

$x = \sqrt[3]{2400} \approx$ (13.4 feet)

$\sqrt[3]{2400}$, $\frac{3}{4}\sqrt[3]{2400}$, $\frac{1200}{(\sqrt[3]{2400})^2}$

#5



$\sqrt{x^4 + (.5)^2} \cdot (\$8) + (6-x) \cdot (\$6)$

$C(x) = 8\sqrt{x^2 + \frac{1}{4}} + 36 - 6x$

$8 \cdot \frac{1}{2}\left(x^2 + \frac{1}{4}\right)^{-\frac{1}{2}} \cdot (2x) - 6 = 0$

$\frac{8x}{\sqrt{x^2 + \frac{1}{4}}} - \frac{6\sqrt{x^2 + \frac{1}{4}}}{\sqrt{x^2 + \frac{1}{4}}} = 6$

$\frac{8x}{\sqrt{x^2 + \frac{1}{4}}} = 6$

$3\sqrt{x^2 + \frac{1}{4}} = 4x$

$9\left(x^2 + \frac{1}{4}\right) = 16x^2$

$\frac{3}{2}\sqrt{4x^2 + 1} = 4x$

$9x^2 + \frac{9}{4} = 16x^2$

$7x^2 = \frac{9}{4}$ $x \approx .567$

$x^2 = \frac{9}{28}$ $x = \frac{3}{\sqrt{28}} = \frac{3}{2\sqrt{7}}$ miles directly