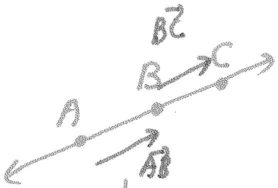
Notes:

- parallel vectors:

If $\vec{a}$ and $\vec{b}$ are parallel, then $\vec{a} = k \cdot \vec{b}$ where $k \in \mathbb{R}$.

ex) Show that $A(-1, 2, 3)$, $B(4, 0, -1)$ and $C(14, -4, -9)$ are collinear.



Show

$\Rightarrow \vec{AB} = k \cdot \vec{BC}$

$$\vec{AB} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ -4 \end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix} 14 \\ -4 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ -8 \end{pmatrix}$$

$\Rightarrow \vec{AB} = k \cdot \vec{BC}$

$$\begin{pmatrix} 5 \\ -2 \\ -4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 10 \\ -4 \\ -8 \end{pmatrix}$$

$k = \frac{1}{2}$.

$\therefore$ A, B, and C are collinear.

- The Scalar product (Dot product) of two vectors.

$$\boxed{\vec{a} \cdot \vec{b} \neq \vec{a} \times \vec{b}}$$

dot product $\quad$ cross product

$\vec{a} = a_1 i + a_2 j + a_3 k \qquad \vec{b} = b_1 i + b_2 j + b_3 k$.

$$\Rightarrow \boxed{\begin{aligned} \vec{a} \cdot \vec{b} &= (a_1 i + a_2 j + a_3 k) \cdot (b_1 i + b_2 j + b_3 k) \\ &= a_1 b_1 + a_2 b_2 + a_3 b_3 \end{aligned}}$$

ex) $\vec{a} = -i + 3j$ and $\vec{b} = -i + 2j$

$\Rightarrow$ Find $\vec{a} \cdot \vec{b}$.

Solution: $(-i + 3j) \cdot (-i + 2j)$

$= (-1)(-1) + (3)(2) = 1 + 6 = \boxed{7}$

ex) $\vec{a} = 2i - 3j + k$, and $\vec{b} = -3i + 5k + 2j$

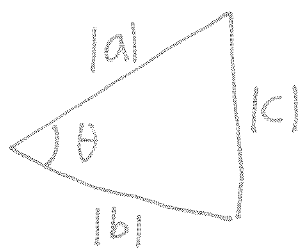
$\Rightarrow$ Find $\vec{a} \cdot \vec{b}$.

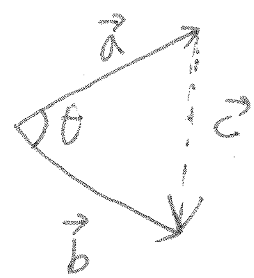
Solution: $(2i - 3j + k) \cdot (-3i + 2j + 5k)$

$= (2)(-3) + (-3)(2) + (1)(5) = -6 - 6 + 5 = \boxed{-7}$

⊙ Angle between two vectors:

$\boxed{a \cdot b = |a||b| \cos\theta}$ $\Rightarrow$ $\cos\theta = \dfrac{a \cdot b}{|a||b|}$.



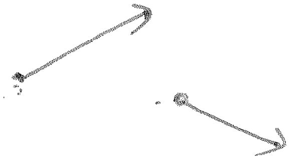


$\vec{a} + \vec{c} = \vec{b}$

$\Rightarrow$ $\boxed{c^2 = a^2 + b^2 - 2a \cdot b \cos\theta \iff |\vec{a}||\vec{b}| \cos\theta = \vec{a} \cdot \vec{b}}$

ex) When $\vec{a} = -i + 3j$ and $\vec{b} = -i + 2j$,

Find the angle between two vectors.



Solution: $\vec{a} \cdot \vec{b} = 7$

$|a| = \sqrt{(-1)^2 + (3)^2} = \sqrt{10}$

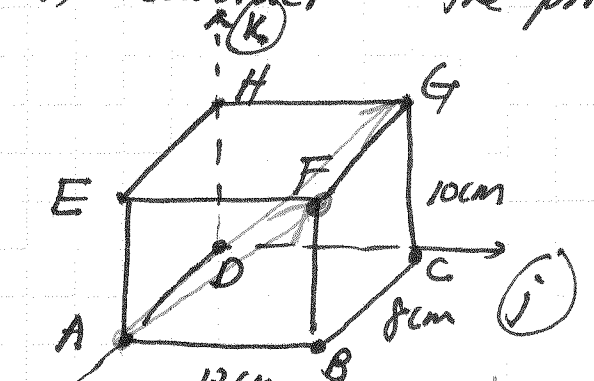
$|b| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$

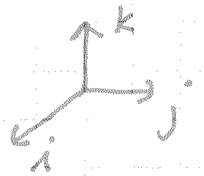
$\cos\theta = \dfrac{7}{\sqrt{10}\sqrt{5}}$ $\Rightarrow$ $\theta = \cos^{-1}\left(\dfrac{7}{\sqrt{10}\sqrt{5}}\right) \approx \underline{8.13^\circ}$

$\approx \underline{.142 \text{ rad}}$

Exit Slip:

1) Consider the prism shown.





a) Find the vectors $\vec{AF}$ and $\vec{AG}$

$\vec{AF} = 12j + 10k$

$\vec{AG} = -8i + 12j + 10k$

b) Find the angle between $\vec{AF}$ and $\vec{AG}$

2) Two vectors are defined as $\vec{a} = 2i + xj$ and $\vec{b} = i - 4j$.

a) Find the value of x if the two vectors are perpendicular.

b) Find the value(s) of x if the two vectors are parallel.