IB Math HL 1 Power Rule WS **Name:** ______________ **Period:** ________

Work with your group.

Part I: Finding a pattern in the Derivatives of polynomials.

1. Use the definition of derivative, find the derivatives , $f'(x)$, of the following polynomials.

$f(x)$	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	$f'(x)$
$f(x)=x$	Work:	$f'(x)=$ 1
$f(x)=x^2$	Work:	$f'(x)=$ $2x$
$f(x)=x^3$	Work: $\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$ $=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$ $=\lim_{h\to 0}\frac{\cancel{h}[3x^2+3xh+h^2]}{\cancel{h}}=\boxed{3x^2}$	$f'(x)=$ $3x^2$
$f(x)=x^4$	Without working through the definition, predict the derivative of $f(x)=x^4$.	$f'(x)=$ $4x^3$
$f(x)=x^n$	**Observing the patterns of above, make a conjecture of the derivative of** $f(x)=x^n$. ***(Conjecture: unproven mathematical theorem based on patterns)***	$f'(x)=$ nx^{n-1}
$f(x)=5$	Now using the above conjecture, predict the derivative of $f(x)=5$. $=5\cdot x^0$	$f'(x)=$ 0

2. Conclusion: The conjecture you made for derivative of $f(x)=x^n$ is called the **Power rule.** Explain the power rule.

$$\boxed{\frac{df}{dx}=f'(x)=nx^{n-1}}$$

$1! = 1$

$0! = 1$

$\frac{n!}{(n-1)!} = n$

$= \frac{n(n-1)(n-2)\cdots 1}{(n-1)\cancel{(n-2)}\cdots 1}$

Part II: Prove of the power rule of Derivative of a polynomial.

1. Using the binomial expansion theorem, expand $f(x) = (x+h)^n$.

Work: $f(x) = x^n + \left[\frac{n!}{1!(n-1)!}\right]x^{n-1}\cdot h + \left[\frac{n!}{2!(n-2)!}\right]x^{n-2}\cdot h^2 \cdots h^n$

2. Evaluate $f'(x) = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = (n-1)x^n$.

Work: $\frac{df}{dx} = f'(x) = \lim_{h\to 0}\frac{[\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 \cdots h^n] - \cancel{x^n}}{h}$

$= \lim_{h\to 0}\frac{\cancel{h}\left[n\cdot x^{n-1} + \frac{n(n-1)}{2}x^{n-2}\cdot h \cdots h^{n-1}\right]}{\cancel{h}} = n x^{n-1}$

Part III: Additional Rules:

Multiplication by a Scalar:

If $f(x) = a\cdot g(x)$, then $f'(x) = ag'(x)$

Addition and Subtraction:

If $h(x) = f(x) \pm g(x)$, then $h'(x) = f'(x) \pm g'(x)$

Part VI: Finding the derivatives using the power rule and Additional rules.

Use the rules you have learned in this activity to find y' $(\frac{dy}{dx})$. *You may need to rewrite some expressions in the form of* x^n.

1. $y = 3x^4 - 7x + 1$

$\Rightarrow \frac{dy}{dx} = y'(x) = \boxed{12x^3 - 7x^0 + 0}$

$= 12x^3 - 7$

2. $y = 8x^9 + 4x^5 - 3x^2 + 2$

$\Rightarrow \frac{dy}{dx} = y'$

$= \boxed{72x^8 + 20x^4 - 6x}$

3. $y = \frac{1}{x^4} = x^{-4}$

$\frac{dy}{dx} = y' = (-4)x^{-5}$

$= \boxed{\frac{-4}{x^5}}$

4. $y = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$

$\frac{dy}{dx} = y' = -\frac{1}{3}x^{-\frac{4}{3}} = \frac{-1}{3x^{\frac{4}{3}}} = \frac{-1}{3x^{\frac{3}{3}}\cdot x^{\frac{1}{3}}}$

$= \boxed{\frac{-1}{3x\sqrt[3]{x}}}$

5. $y = \frac{x^5 - 4}{x^3} = \frac{x^5}{x^3} - \frac{4}{x^3}$

$= x^2 - 4x^{-3}$

$\frac{dy}{dx} = y' = 2x + 12x^{-4}$

$= 2x + \frac{12}{x^4}$

6. $y = (2-7x)^2$