IB Math HL2: **Name: ______________ Period: _______**

Power Series

Warm up: Approximate the sum of the alternating series by using the first six terms. Include the remainder. Give the final answer rounding to 5 decimal places.

a_{n+1}.

a_7.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{3}{n!}\right)$$

$S_6 = (3) - \frac{3}{2!} + \frac{3}{3!} - \frac{3}{4!} + \frac{3}{5!} - \frac{3}{6!} \approx \frac{91}{48}$

$a_7 = \frac{3}{7!}$

$\boxed{\frac{91}{48} - \frac{1}{1680} \quad S \le \frac{91}{48} + \frac{1}{1680}}$

Part I:

- **Definition of Power Series:**

If x is a variable, then an infinite series of the form:
1) If $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 ......a_n x^n.....$ **is called a power series centered at x=0**
2) If $\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 ......a_n(x-c)^n.....$ **is called a power series centered at x=c.**

Definition of Radius and Interval of Convergence:

If a power series is viewed as a function, $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$**, the series possibly converges or diverges depend on the set values of x . This set values of x is called the interval of convergence and the half of the interval of convergence is called the Radius of convergence.**

- **Convergence of a power series Theorem:**

One of the following is true for a power series which is centered at c
1) The Series converges only at x=c.
2) The Series converges absolutely for $\lvert x-c \rvert < R$ **where** $R \in \mathbb{R}^+$**. (R value is called the radius of convergence)**
3) The Series converges absolutely for all x.

Example 1) Find the radius of convergence for $\sum_{n=10}^{\infty} n!x^n$

Ratio test.

$a_{k+1} = (k+1)!\, x^{k+1} \qquad a_k = k!\, x^k$

$\Rightarrow \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{(k+1)!\, x^{k+1}}{k!\cdot x^k} = \lim_{k\to\infty} \frac{(k+1)\cancel{(k!)}\cancel{x^k}\cdot x}{\cancel{k!}\cdot\cancel{x^k}} \lim_{k\to\infty}(k+1)\cdot x$

$= \infty > 1 \Rightarrow \sum_{n=10}^{\infty} n!x^n$ diverges. ~~Except~~ $x=0$

$\Rightarrow x=0$.

Example 2) Find the radius of convergence for $\sum_{n=0}^{\infty} 3(x-2)^n$

Ratio test.

$$\lim_{k\to\infty} \frac{3(x-2)^{k+1}}{3(x-2)^k} = \lim_{k\to\infty} \frac{\cancel{3}\,\cancel{(x-2)^k}(x-2)}{\cancel{3}\,\cancel{(x-2)^k}} = (x-2)$$

$$|x-2|<1 \Rightarrow -1<x-2<1$$

$$\Rightarrow 1<x<3 \Rightarrow \text{Radius}: 1.$$

Example 3) Find the interval of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n(x+1)^n}{2^n}$ including the end points.

Ratio test:

$$\lim_{k\to\infty} \left| \frac{(-1)^{k+1}\cdot(x+1)^{k+1}}{2^{k+1}} \right| \cdot \left| \frac{2^k}{(-1)^k(x+1)^k} \right|$$

$$= \lim_{k\to\infty} \left(\frac{\cancel{(x+1)^k}(x+1)}{\cancel{2^k}\cdot 2} \right)\left(\frac{\cancel{2^k}}{\cancel{(x+1)^k}} \right) = \frac{x+1}{2}.$$

Radius: $\left|\frac{x+1}{2}\right|<1 \Rightarrow -2<x+1<2 \Rightarrow \boxed{\text{Radius}: 2}$

$$\Rightarrow -3<x<1$$

End points: ① when $x=-3$.

$$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n(-3+1)^n}{2^n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(-2)^n}{2^n}$$

$$= \sum_{n=0}^{\infty} (-1)^n\cdot(-1)^n\cdot\frac{\cancel{2^n}}{\cancel{2^n}}$$

$$= \sum_{n=0}^{\infty} (1)^n \Rightarrow G \text{ series } r=1$$

diverges.

② when $x=1$.

$$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n\cancel{(1+1)^n}}{\cancel{2^n}}$$

$$= \sum_{n=0}^{\infty} (-1)^n = 1-1+1-1\cdots$$

$\Rightarrow$ G series $r=|-1|$

diverges.

$\therefore$ Interval

$-3<x<1 \Rightarrow (-3, 1)$

IB Math 3: **Name:** Sample Work **Period:** ______

Estimation of Alternating Series and Power Series review

Practice:

Find the radius of convergence and the interval of convergence for the following series:

1) $\sum_{n=1}^{\infty}\left(\frac{(x-3)^n}{n}\right)$

Ratio test:

$$\lim_{n\to\infty}\left(\frac{(x-3)^{n+1}}{n+1}\right)\left(\frac{n}{(x-3)^n}\right)$$

$$=\lim_{n\to\infty}\left(\frac{\cancel{(x-3)^n}(x-3)}{n+1}\right)\left(\frac{n}{\cancel{(x-3)^n}}\right)$$

$$=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)(x-3)=\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)(x-3)$$

$$= x-3$$

$|x-3|<1 \Rightarrow -1<x-3<1$

$\boxed{R=1}$ $\boxed{2<x<4}$

End points:

$x=2$, $\sum \frac{(2-3)^n}{n}$

$=\sum \frac{(-1)^n}{n} \Rightarrow$ Alt. Series test $\lim_{n\to\infty}\frac{1}{n}=0$

$\left(\frac{1}{n}\right)'=\frac{-1}{n^2}<0$ decreasing.

Conditionally converges.

$x=4$ $\sum \frac{(4-3)^n}{n}=\sum\frac{1}{n}$

Diverges by p series $p=1$.

$\therefore$ $\boxed{2\le x<4}$

$[2,4[$ OR $[2,4)$.

2) $\sum_{n=0}^{\infty}\left(\frac{(-3)^n x^n}{\sqrt{n+1}}\right)$

Attached.

3) $\sum_{n=1}^{\infty}\left(\frac{(-1)^n x^{2n-1}}{(2n-1)!}\right)$

4) $\sum_{n=1}^{\infty}\left(\frac{(-1)^n (2x+3)^n}{n\ln n}\right)$