practice W.S solutions.

1. $\ln y = 2x + 6\ln(2x-1) - 2\ln(x^3+5) - \ln(4-7x)$

$$\frac{dy}{dx} = y\left[2 + \frac{12}{2x-1} - \frac{6x^2}{x^3+5} + \frac{7}{4-7x}\right]$$

$$= \left[\frac{e^{2x}(2x-1)^6}{(x^3+5)^2(4-7x)}\right]\left[2 + \frac{12}{2x-1} - \frac{6x^2}{x^3+5} + \frac{7}{4-7x}\right]$$

2. $\ln y = 2x\ln(\tan^{-1}x)$

$$\frac{dy}{dx} = y\left[2\ln(\tan^{-1}x) + \frac{2x}{(\tan^{-1}x)(1+x^2)}\right]$$

$$= (\tan^{-1}x)^{2x}\left[2\ln(\tan^{-1}x) + \frac{2x}{(\tan^{-1}x)(1+x^2)}\right].$$

3. $\ln y = \sqrt{x}\cdot\ln x$

$$\frac{dy}{dx} = y\left[\frac{1}{2\sqrt{x}}\cdot\ln x + \frac{\sqrt{x}}{x}\right] = \left[x^{\sqrt{x}}\right]\cdot\left[\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}\right]$$

4. $\ln y = \sin x\cdot\left[3\ln(2x-1) - 4\ln(x^2+1)\right]$

$$\frac{dy}{dx} = y\left[\cos x\ln\left(\frac{(2x-1)^3}{(x^2+1)^4}\right) + \sin x\left[\frac{6}{2x-1} - \frac{8x}{x^2+1}\right]\right]$$

$$= \left(\frac{(2x-1)^3}{(x^2+1)^4}\right)^{\sin x}\cdot\left[\cos x\ln\left(\frac{(2x-1)^3}{(x^2+1)^4}\right) + \frac{6\sin x}{2x-1} - \frac{8x\sin x}{x^2+1}\right]$$

5. $\frac{dy}{dx} = \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}} = \frac{\frac{1}{3}}{\sqrt{1-\frac{x^2}{9}}} = \frac{\frac{1}{\cancel{3}}}{\frac{\sqrt{9-x^2}}{\cancel{3}}} = \boxed{\frac{1}{\sqrt{9-x^2}}}$

6. $\frac{dy}{dx} = \boxed{\ln 5 \cdot 5^x \cos^{-1} x - \frac{5^x}{\sqrt{1-x^2}}}$

7. $\frac{dy}{dx} = \boxed{3\left[\tan^{-1}(5x)\right]^2 \cdot \frac{5}{(1+25x^2)}}$

8. $\frac{dy}{dx} = \boxed{\frac{1}{2\sqrt{\sec^{-1} x}} \cdot \frac{1}{|x|\sqrt{x^2-1}}}$

9. $\frac{d}{dx}\, x^2 - xy + y^2 = \frac{d}{dx}(4)$

$\Rightarrow 2x - y - x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx}\left[2y - x\right] = y - 2x$

$\frac{dy}{dx} = \frac{y-2x}{2y-x}.$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{\left[\frac{dy}{dx} - 2\right]\left[2y-x\right] - \left[(y-2x)\left(2 \cdot \frac{dy}{dx} - 1\right)\right]}{(2y-x)^2}$

$= \frac{\left[\left(\frac{y-2x}{\cancel{2y-x}}\right)\cdot(\cancel{2y-x}) - 2(2y-x) - 2(y-2x)\left(\frac{y-2x}{2y-x}\right) + (y-2x)\right]}{(2y-x)^2}$

$= \frac{\cancel{y} - \cancel{2x} - \overset{-2y}{\cancel{4y}} + \cancel{2x} - \left[2(y-2x)\left(\frac{y-2x}{2y-x}\right)\right] + \cancel{y} - 2x}{(2y-x)^2} = \frac{-2y - 2x - 2(y-2x)(y-2x)\left(\frac{1}{2y-x}\right)}{(2y-x)^2} \Rightarrow$

$\Uparrow$

$\boxed{\frac{-2(y+x)(2y-x) - 2(y-2x)^2}{(2y-x)^3}}$

10. $\frac{d}{dx} x^2 + 4y^2 = \frac{d}{dx} 17$

$\Rightarrow 2x + 8y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{8y} = \frac{-x}{4y}.$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(4y) + x \cdot 4 \cdot \frac{dy}{dx}}{16y^2}$

$= \frac{\left[-4y + \cancel{4}x \left(\frac{-x}{\cancel{4}y}\right)\right] \cdot y}{(16y^2) \cdot y}$

$= \boxed{\frac{-4y^2 - x^2}{16y^3}}$

11. $3x^2y - 2xy^2 = 1$

1) Find $\frac{dy}{dx}$

$6xy + 3y^2 \cdot \frac{dy}{dx} - 2y^2 - 4xy \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx}[3y^2 - 4xy] = 2y^2 - 6xy$

$\frac{dy}{dx} = \frac{2y^2 - 6xy}{3y^2 - 4xy}.$

2) Find the value of y when $x = 1$

$3y - 2y^2 = 1 \Rightarrow 2y^2 - 3y + 1 = 0$

$2y \quad -1$

$y \quad -1$

$y = \frac{1}{2}, \quad y = 1$

3) $\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2-6}{3-4} = \frac{-4}{-1} = 4$

$\perp$ slope : $\frac{-1}{4} \Rightarrow \boxed{y - 1 = \frac{-1}{4}(x-1)}$

$\left.\frac{dy}{dx}\right|_{(1,\frac{1}{2})} = \frac{(2)(\frac{1}{2})^2 - \cancel{(6)(\frac{1}{2})}}{3(\frac{1}{2})^2 - 4(\frac{1}{2})}$

$= \frac{\frac{1}{2} - 3}{\frac{3}{4} - 2} = 2$

$\perp$ slope : $\frac{-1}{2} \Rightarrow \boxed{y - \frac{1}{2} = \frac{-1}{2}(x-1)}$