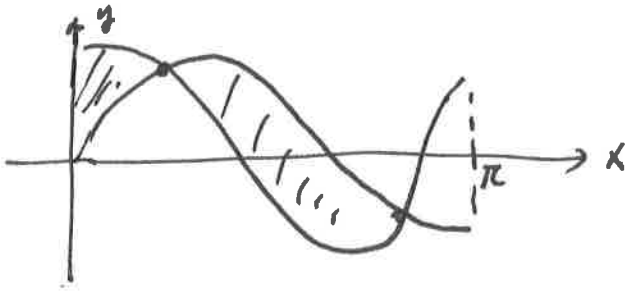
IB Math HL1
Chapter 22 Quiz Form C

Name key
Date _____ Period _____

CALCULATOR is Okay! Show all work. Box your final answer.

1. Given the x-axis, $f(x) = \cos 2x$ and $g(x) = \sin 2x$.

a) Sketch the graphs in $x \in [0, \pi]$. [2]



a) Algebraically find the first two intersections of x values in $[0, \pi]$. Show your work and give your answer in exact. [2]

$\cos 2x = \sin 2x$

$\cos 2x - \sin 2x = 0$

$2x = \frac{\pi}{4}, \frac{5\pi}{4} \Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}$.

c) Find the area between the curves. Show your work in two integrals and give your answer in 3 significant figures. [3]

$$A = \int_0^{\frac{\pi}{8}} (\cos 2x - \sin 2x)\,dx + \int_{\frac{\pi}{8}}^{\frac{5\pi}{8}} (\sin 2x - \cos 2x)\,dx$$

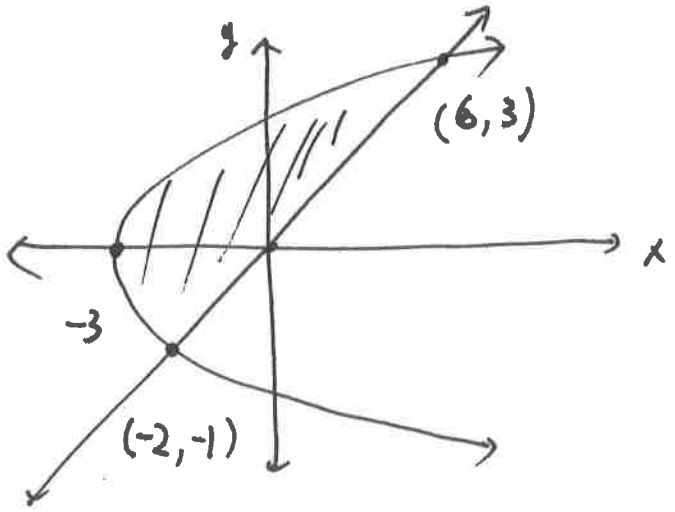
$$= \left[\frac{1}{2}\sin 2x + \frac{1}{2}\cos 2x\right]_{x=0}^{x=\frac{\pi}{8}} + \left[-\frac{1}{2}\cos 2x - \frac{1}{2}\sin 2x\right]_{x=\frac{\pi}{8}}^{x=\frac{5\pi}{8}}$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right] - \frac{1}{2}[\cos 0] - \frac{1}{2}\left[\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\right] + \frac{1}{2}\left[\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right]$$

$$\approx \boxed{1.62.}$$

2. Given $x = y^2 - 3$ and $x = 2y$

a) Sketch the graphs of these two curves showing the intersection coordinates. [3]



$y^2 - 3 = 2y$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$y = -1 \quad y = 3$

$x = -2 \quad x = 6$

b) Find the area between two curves. Show your work and give your answer in exact. [3]

$$A = \int_{-1}^{3} (2y - (y^2 - 3))\,dy$$

$$= \left[y^2 - \frac{1}{3}y^3 + 3y \right]_{x=-1}^{x=3}$$

$$= \left[3^2 - \frac{1}{3}(3)^3 + 9 \right] - \left[1 + \frac{1}{3} - 3 \right] = \boxed{\frac{32}{3}}$$

3. A particle P moves in a straight line with velocity function $v(t) = 3t^2 - 6t$ cm/sec, $t \geq 0$.
The particle is initially 2 cm to the right of O.

a. Write a formula for the position function $s(t)$. [2]

$$S(t) = \int (3t^2 - 6t)\,dt = t^3 - 3t^2 + C \qquad C = 2$$

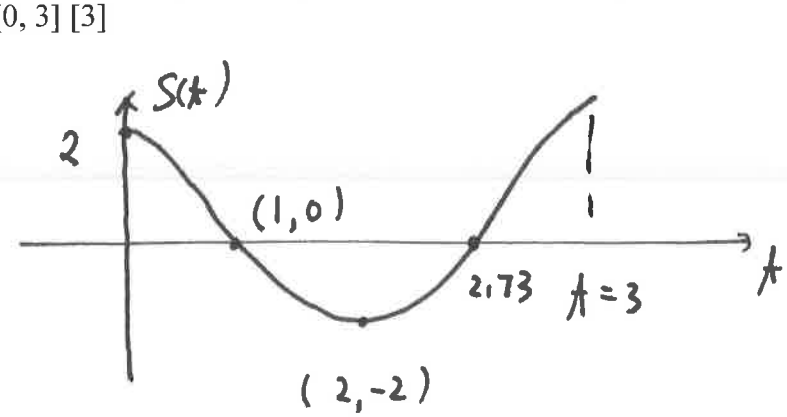
$$\boxed{S(t) = t^3 - 3t^2 + 2}$$

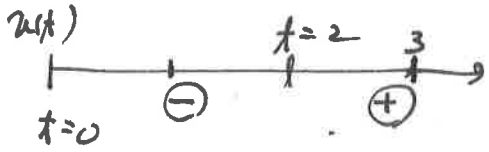
b. Find the time when the particle changes its direction. [1]

$3t^2 - 6t = 0$

$3t(t-2) = 0$ $\quad t = 0$ $\quad \boxed{t = 2}$ ← Change the direction.

c. Sketch the graph of the position function showing clearly the local max, local min, and x-intercept(s) in [0, 3] [3]





d Find the displacement of P after 3 seconds. Show your work and give your answer in exact. [2]

$$\text{displacement} = \int_0^3 (3t^2 - 6t)\,dt = S(3) - S(0)$$

$$= 2 - 2 = \boxed{0} \text{ cm}$$

e. Find the total distance of P after 3 seconds. Show your work and gie your answer in exact. [2]

$$\text{Total distance} = \int_0^2 -(3t^2 - 6t)\,dt + \int_2^3 (3t^2 - 6t)\,dt$$

$$= |S(0) - S(2)| + |S(3) - S(2)| = 4 + 4 = 8 \text{ cm}$$