Mathematical Exploration

Modelling Rainfall

Rationale:

I have decided to centre this mathematical exploration around the topic of weather.
To begin, I looked at the stimuli given to me by my teacher. While looking at different ideas, weather was a topic that stood out to me, because never before, had I thought about the mathematical significance in weather. Would it be possible to predict tomorrow's weather using Mathematics? I did some research about the significance in meteorology, but did not find anything very interesting in particular. Nevertheless, I wanted to stick to the topic of weather and decided to relate it to a more personal matter. Having lived in a country like Germany for nearly all my life, one is face to face with rain almost every day of the year. It comes and goes, no matter what. I thus decided to write this exploration about rain, that is falling objects. Is it possible to model rainfall? What would I need to consider? I first decided on the factors, which I wanted to account for in my model and then looked a little closer at rain itself. There are different kinds of rain, and one would have quite different experiences walking through a light drizzle than being outside on the street during a thunderstorm.
I thus decided to explore different sizes of raindrops, first very small drops with diameter $d \leq 0.008cm$ and then larger drops with diameter $d \geq 0.125cm$. In addition, falling objects are not only dependent upon gravity and mass itself, but on air resistance. Raindrops thus experience an upward drag, which must be accounted for when modelling rainfall. The velocity upon impact, thus terminal velocity of the raindrop, and the time it takes the raindrop to reach the ground were determined.
Finally, the model was extended to a parachute jump, which in a way is a very, very large raindrop falling down from the sky. However, there is one essential difference: the parachute itself, as a skydiver will always experience two periods of falling: the free-fall and the slow descent, after the parachute is activated. This must be accounted for when developing a model.

Introduction:

Rain is a phenomenon we come across on a regularly basis in our day-to-day life. But what do we actually know about it? We know it is liquid precipitation and that rain can be of different intensity, such as light rain (a drizzle) or heavy rain (a storm). This exploration aims to develop a model for rainfall.

In order to correctly model rainfall, I decided to first determine the factors related to rainfall, which are thus of importance to the model. As mentioned above, the intensity of rain is never the same. It is a measure of the amount of rain that falls over time, thus measured in the height of the water layer covering the ground in a period of time, millimetres per hour. But not only the intensity of rain matters. In order to determine the other factors, I found it easiest to just think of a falling object, such as a pen, which is dropped at a certain height. What factors will have an impact upon the pen? Gravitational force would be the first thing that springs to mind, but also acceleration and velocity are important factors to consider, as is the distance of the pen from the ground and the time it takes the pen to reach the ground.

First Model (not accounting for air resistance):

To develop the first model taking into account the factors mentioned above, I did some research about the forces upon falling objects and found Newton's Second law of motion, which states that force equals mass times acceleration.[1] This can be written as:

$F = ma$, where F is force, m the mass of the object and a its acceleration.
Newton's Law of Motion can be used to model a raindrop with constant acceleration, that is, ignoring the air resistance. It is known that acceleration, a, is the first derivative of velocity, v, and the second derivative of position, s. This can be written as:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

For a falling object, we can also state that $a = g$. We then get:

$$\frac{dv}{dt} = a = g$$

We thus obtain a differential equation for a raindrop with constant acceleration, for which, in order to solve it, we first need to set some initial conditions: The raindrop is starting at rest from 1000 metres in the air, thus the origin. By doing some research, it was obtained that the constant acceleration of falling objects, on which the only significant force in gravitational force, is $g = 9.81 ms^{-2}$. It can also be stated that, at origin, $v(0) = 0, s(0) = 0, t = 0$.

The differential equation can now be solved using basic integration.

$$\frac{dv}{dt} = g$$

$$dv = gdt$$

$$\int dv = \int gdt$$

$$v = gt + k$$

To find the constant of integration, the initial conditions explained above must be used, thus $v(0) = 0, t = 0$

$$0 = g(0) + k$$

$$k = 0$$

Thus:

$$v = gt$$

We can now use another position equation, $\frac{ds}{dt} = v$, to continue. Again, basic integration is used to solve the differential equation.

[1] "Newton's Three Laws of Motion", Astronomy 162 Stars, Galaxies, and Cosmology,
<http://csep10.phys.utk.edu/astr161/lect/history/newton3laws.html>, 1 May 2010
[2] "Acceleration of Gravity and Newton's Second Law of Motion", The Engineering ToolBox,
<http://www.engineeringtoolbox.com/acceleration-gravity-d_340.html>, 3 May 2010

$v = gt$

$v = \frac{ds}{dt}$

$\frac{ds}{dt} = gt$

$ds = gt\,dt$

$\int ds = \int gt\,dt$

$s = \frac{g}{2}t^2 + k$

To determine the constant of integration, the initial conditions have to be used again, thus $s(0) = 0, t(0) = 0$. Thus,

$0 = \frac{g}{2}(0)^2 + k$

$k = 0$

We now get:

$s = \frac{g}{2}t^2$

As stated above, $s = 1000m$ and $g = 9.81ms^{-2}$. These values can be substituted in the above equation to solve for time, t:

$1000 = \frac{9.81}{2}t^2$

$2000 = 9.81t^2$

$t^2 = \frac{2000}{9.81}$

$t = \sqrt{\frac{2000}{9.81}}$

$t = 14.278$

To determine the velocity of the raindrop upon arriving on the ground we can use the first equation,

$v = gt$

$v = 9.81 \times 14.278$

$v = 140.0714$

We have now modelled a raindrop falling from a given distance, 1000 metres, which experience constant acceleration, $g = 9.81ms^{-2}$. As aimed, the results obtained show the time, how long it takes the raindrop to reach the ground, and its velocity upon reaching the ground. As shown above it would take the raindrop approximately 14.278 seconds and it would hit the ground at around 140.0714 metres per second.

The value of the velocity seems rather big. In fact, this model is an unrealistic one, as the raindrop was said to fall form a distance of one kilometre. In addition, only gravitational force was taken into account. In reality, however, the raindrop is subject to air resistance and thus dragged upwards.

However, when observing the constant acceleration model, it can be said that it will work for any size of raindrop, since neither acceleration, nor velocity is dependent on mass.

When taking a closer look at the equation for acceleration, $\frac{dv}{dt} = g$, we find that it is not possible to determine the terminal velocity of the raindrop, as $g = 9.81ms^{-2}$ and acceleration can thus never be zero. This means, that the raindrop will not stop accelerating, which in reality, is not the case.[3]

Second Model (small raindrops, accounting for air resistance):

Having found this first model for rainfall, we now want to develop a more realistic one, thus taking into account the air resistance, which is the force that acts against anything that moves through the atmosphere or air. The amount of air resistance is dependent upon a variety of factors, most important are the speed of the object and its cross sectional area.[4] Thus both increased speed and cross sectional area of a moving object leads to an increase in the amount of air resistance. To account for the air resistance encountered by air drops one must first find some further information about the upward drag. Research shows that for very small drop with diameter $d \leq 0.008cm$ experience a drag force which is proportional to the velocity. Given that force is proportional to acceleration we get:

$F_{drag} \propto a_{drag} \propto v$

Thus,

$a_{drag} = kv$

Experiment shows that $k = 12.2s^{-1}$.

As stated above, when not accounting for air resistance, is constant, $g = 9.81ms^{-2}$.

In order to find a new differential equation for this model, we now need to account for the constant acceleration and the upward drag. Acceleration is thus given by:

$a = \frac{dv}{dt}$

$\frac{dv}{dt} = g - kv$

Using this differential equation we can easily find the terminal velocity of the raindrop. Terminal velocity of a falling object is a constant velocity reached when there is no resultant acceleration. acceleration is thus zero. We can thus obtain the terminal velocity from the model:

$0 = g - kv$

$g = kv$

$v = \frac{g}{k}$

[3] Emerson, Brandon. "Raindrop Modeling", <bpcmr.people.wm.edu/Raindrops.doc>, 28 April 2010

[4] "The Forces at Work – Gravity", < http://fiden-2.phys.uaf.edu/211.fall2000/web.projects/vlad%20/paverman/forces.htm>, 22 May 2010

Substituting in the values for g and k we get:

$$v = \frac{9.81}{12.2}$$
$$v = 0.804$$

As we continue working on the model, thus integrating it, we can prove the above result by graphing the equation found for velocity. Also, as we find the time it takes the raindrop to reach the floor, the velocity equation, which takes time into account, should give us the same result as above.

To continue, we now integrate the differential equation:

$$\int \frac{dv}{g - kv} = \int dt$$

$$\left(-\frac{1}{k}\right)\ln(g - kv) = t + a$$

$$\ln(g - kv) = -kt + a$$

$$g - kv = e^{-kt+a}$$

$$g - kv = e^{-kt} \times e^{a}$$

e^{a} is a constant and can thus be written as b to simplify the equation. We now get:

$$g - kv = b \times e^{-kt}$$

To obtain the constant of integration, b, we consider the initial conditions, which are the same as in the first model, $v(0) = 0, t = 0$. Thus,

$$g - k(0) = b \times e^{-k(0)}$$
$$g = b$$

Plugging in the constant of integration and solving for the velocity, v we get:

$$g - kv = ge^{-kt}$$
$$g - ge^{-kt} = kv$$
$$v = \frac{g - ge^{-kt}}{k}$$
$$v = \frac{g}{k}(1 - e^{-kt})$$

We can now graph the equation above to prove that the terminal velocity, which was determined earlier, is correct. The graph should, at one point, have a gradient of zero. Thus, once the graph reaches a horizontal plateau, we can determine the terminal velocity.

Point at which the gradient of the curve is firstly equal to zero,
x=1, y=0.804

$$y = \frac{9.81}{12.2}(1 - e^{-12t})$$

v
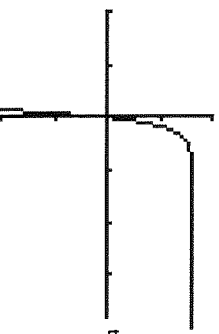
```
WINDOW
Xmin=-1
Xmax=2
Xscl=.5
Ymin=-1
Ymax=1
Yscl=.5
Xres=1
```

X	Y1
0	0
1	.8049
2	.8041
3	.8041
4	.8041
5	.8041
6	.8041

Y1=(9.81/12.2)*...

As seen above, the graph is horizontal, and the point at which the gradient is equal to zero is reached very fast, within a minute. It can thus be assumed that the droplets travel with terminal velocity for almost the entire fall. This seems rather odd at first, why would a falling body experience zero acceleration for such a long time? This is due to the retarding force of air resistance, which exists because air molecules collide into a falling body and create an upward force opposite gravity. Eventually, this upward drag balances the weight of the falling object, which will thus continue to fall at constant, terminal velocity.[5]

As before, we now use a second position equation, $\frac{ds}{dt} = v$. Thus:

$$v = \frac{g}{k}(1 - e^{-kt})$$
$$\frac{ds}{dt} = \frac{g}{k}(1 - e^{-kt})$$
$$ds = \frac{g}{k}(1 - e^{-kt})dt$$

We can now integrate this problem:

[5] Elert, Glenn, "Speed of a Skydiver (Terminal Velocity)", The Physics Factbook http://hypertextbook.com/facts/JianHuang.shtml, 10 May 2010

$$\int ds = \int \frac{g}{k}(1-e^{-kt})dt$$
$$\Rightarrow s = \frac{g}{k}\left(t + \frac{1}{k}e^{-kt}\right) + c$$

Solving for the constant of integration, c, we use the initial conditions, $s(0)=0, t(0)=0$. We then get:

$$0 = \frac{g}{k}\left(0 + \frac{1}{k}\right) + c$$
$$\Rightarrow c = -\frac{g}{k^2}$$

Plugging in the constant of integration:

$$s = \frac{g}{k}\left(t + \frac{1}{ke^{kt}}\right) - \frac{g}{k^2}$$
$$s = \frac{g}{k}\left(t + \frac{1}{ke^{kt}}\right) - \frac{g}{k^2}$$

We can now substitute in the known values, $s=1000, g=9.81, k=12.2$, to obtain:

$$1000 = \frac{9.81}{12.2}\left(t + \frac{1}{12.2e^{12.2t}}\right) - \frac{9.81}{12.2}$$

Instead of solving the equation algebraically, it is better to just graph the equation $y = \frac{9.81}{12.2}\left(t + \frac{1}{12.2e^{12.2t}}\right) - \frac{9.81}{12.2}$ and find the intersection with the equation $y=1000$. This can be done on the graphing display calculator. However, to avoid complicated fractions it is best to first simplify:

$$12200 = 9.81t + \frac{9.81}{12.2e^{12.2t}} - \frac{9.81}{12.2}$$

We thus graph the two equations and find the point of intersection of the two graphs.

$y=12000$

Point of intersection of the two graphs: (1243.709, 12200)

$y = 9.81t + \frac{9.81}{12.2e^{12.2t}} - \frac{9.81}{12.2}$

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Intersection
X=1243.7109  Y=12200
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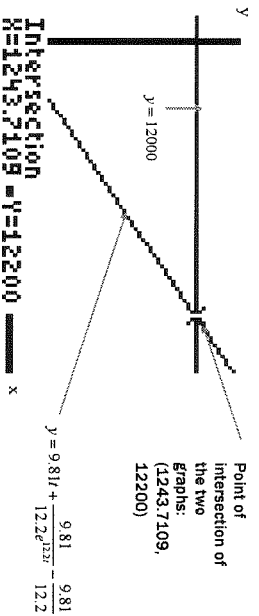
```
WINDOW
Xmin=-100
Xmax=1500
Xscl=1
Ymin=-300
Ymax=15000
Yscl=1
Xres=1
```

We thus get obtain that, at y=12200, x=1243.711. This number now needs to be divided by sixty, to determine the time in minutes it will take the raindrop to reach the ground.

$$\frac{1243.711}{60} = 20.729$$

It thus takes the raindrop approximately 21 minutes to reach the ground. Having found the time, we can now determine the raindrop's velocity upon reaching the ground. We found that

$$v = \frac{g}{k}(1 - e^{-kt})$$

Thus:

$$v = \frac{9.81}{12.2}(1 - e^{-12.2 \cdot 20.729})$$
$$v = 0.804 m/s$$

As mentioned above, the result is the same as the terminal velocity.

Third model (larger drops, accounting for air resistance):

We will now again extend the model of rainfall. It is unreasonable to assume that all droplets are smaller than 0.008cm. We will thus consider larger raindrops with a diameter $d \geq 0.125cm$. To correctly model the rainfall of the larger drops, we will immediately account for the upward drag, which in this case, experiment shows, is proportional to velocity squared and the constant of proportionality is $c = 0.0097$. We can thus alter the model for smaller raindrops according to the new conditions. Thus:

$$\frac{dv}{dt} = g - kv^2$$

Again, we will integrate to continue:

$$\int \frac{dv}{g - kv^2} = \int dt$$

The integral on the left seems rather difficult, as we need to use partial fractions to integrate. Using partial fractions, we know that:

$$\frac{1}{g-kv^2} = \frac{1}{2\sqrt{g}\times\left(v\sqrt{k}+\sqrt{g}\right)} - \frac{1}{2\sqrt{g}\times\left(v\sqrt{k}-\sqrt{g}\right)}$$

We can now use this to integrate, and, as g is a constant, we can rewrite it as:

$$\int\frac{dv}{2\sqrt{g}\times\left(v\sqrt{k}+\sqrt{g}\right)} - \int\frac{dv}{2\sqrt{g}\times\left(v\sqrt{k}-\sqrt{g}\right)} = \int dt$$

$$\frac{1}{2\sqrt{g}}\times\int\frac{dv}{k\cdot v+\sqrt{g}} - \frac{1}{2\sqrt{g}}\times\int\frac{dv}{v\sqrt{k}-\sqrt{g}} = \int dt$$

Integrating:

$$\frac{1}{2\sqrt{g}}\times\left\{\frac{\ln\left|v\sqrt{k}+\sqrt{g}\right|}{\sqrt{k}} - \frac{\ln\left|v\sqrt{k}-\sqrt{g}\right|}{\sqrt{k}}\right\} = t$$

$$\frac{1}{2\sqrt{g}}\times\left\{\frac{\ln\left|v\sqrt{k}+\sqrt{g}\right| - \ln\left|v\sqrt{k}-\sqrt{g}\right|}{\sqrt{k}}\right\} = t$$

Using the laws of logs, we get:

$$\frac{1}{2\sqrt{g}}\times\frac{\ln\left|\dfrac{v\sqrt{k}+\sqrt{g}}{v\sqrt{k}-\sqrt{g}}\right|}{\sqrt{k}} = t$$

$$\ln\left|\frac{v\sqrt{k}+\sqrt{g}}{v\sqrt{k}-\sqrt{g}}\right| = (t+C)\times 2\sqrt{g}\sqrt{k}$$

The initial conditions are $v=0, t=0$, thus $C=0$ and:

$$\ln\left|\frac{v\sqrt{k}+\sqrt{g}}{v\sqrt{k}-\sqrt{g}}\right| = \left(2\sqrt{kg}\right)t$$

Simplifying:

$$e^{(t\sqrt{kg})} = \frac{v\sqrt{k}+\sqrt{g}}{v\sqrt{k}-\sqrt{g}}$$

$$e^{(t\sqrt{kg})}\left(\sqrt{k}v-\sqrt{g}\right) = \sqrt{k}v+\sqrt{g}$$

$$\sqrt{k}v\left(e^{(t\sqrt{kg})}-1\right) = \sqrt{g}\left(e^{(t\sqrt{kg})}+1\right)$$

$$v = \sqrt{\frac{g}{k}}\left(\frac{e^{(t\sqrt{kg})}+1}{e^{(t\sqrt{kg})}-1}\right)$$

We have now found the formula for the velocity. This is a differential equation for the distance s. Simplifying further we get:

$$v = \sqrt{\frac{g}{k}}\left(\frac{e^{(2\sqrt{kg})t}+1}{e^{(2\sqrt{kg})t}-1}\right)$$

$$\frac{ds}{dt} = \sqrt{\frac{g}{k}}\left(\frac{e^{(2\sqrt{kg})t}-1+2}{e^{(2\sqrt{kg})t}-1}\right)$$

$$\frac{ds}{dt} = \sqrt{\frac{g}{k}}\left(1+\frac{2}{e^{(2\sqrt{kg})t}-1}\right)$$

$$\int ds = \sqrt{\frac{g}{k}}\int\left(1+\frac{2}{e^{(2\sqrt{kg})t}-1}\right)dt$$

In order to integrate this fraction we use the substitution:

$$u = e^{(t\sqrt{kg})}$$

$$du = 2\sqrt{kg}\,e^{(t\sqrt{kg})}dt$$

$$\Rightarrow dt = \frac{du}{2\sqrt{kg}\,u}$$

$$s = \sqrt{\frac{g}{k}}t + 2\sqrt{\frac{g}{k}}\int\frac{du}{2\sqrt{kg}\,u(u-1)}$$

$$s = \sqrt{\frac{g}{k}}t + \sqrt{\frac{g}{k^2 g}}\int\frac{du}{u(u-1)}$$

We can now use partial fractions to solve this last integral. We know that $\dfrac{1}{u(u-1)} = \dfrac{1}{(u-1)} - \dfrac{1}{u}$.

Therefore:

$$\int\frac{du}{u(u-1)} = \int\left(\frac{1}{(u-1)}-\frac{1}{u}\right)du$$, which can be written as:

$$\int\left(\frac{1}{(u-1)}\right)dt = \int\frac{dt}{(u-1)} - \int\frac{dt}{u}$$

This can now be integrated.

$$\int\frac{dt}{(u-1)} - \int\frac{dt}{u} = \ln|u-1| - \ln|u|$$

As we used a substitution for u above, we now need to substitute u back in.

$$u = e^{(t\sqrt{kg})}$$

$$\ln\left|e^{(t\sqrt{kg})}-1\right| - \ln\left|e^{(t\sqrt{kg})}\right|$$

We thus obtain:

$$\ln\left|e^{(t\sqrt{kg})}-1\right| - \ln\left|e^{(t\sqrt{kg})}\right|$$

Substituting the part back into the equation above we get a solution for s:

$$s = \left(\sqrt{\frac{g}{k}}\right)t + \sqrt{\frac{g}{k^2 g}}\left(\ln\left|e^{(t\sqrt{kg})}-1\right| - \ln\left|e^{(t\sqrt{kg})}\right|\right)$$

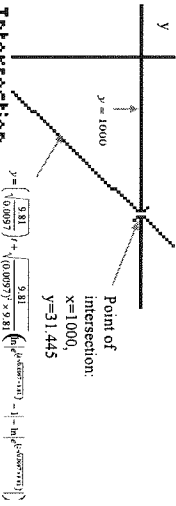We can now substitute in the known values, $s=1000, g=9.81, k=0.0097$, to obtain:

$$1000 = \left(\sqrt{\frac{9.81}{0.0097}}\right)\left[t + \sqrt{\frac{9.81}{(0.0097)^2 \times 9.81}}\left(\ln\left|e^{\left(2\sqrt{0.0097 \cdot 9.81}\right)}\right| - 1\right| - \ln\left|e^{\left(2\sqrt{0.0097 \cdot 9.81}\right)}\right|\right)\right]$$

In order to solve for t, we can graph the two equations $y = 1000$ and

$$y = \left(\sqrt{\frac{9.81}{0.0097}}\right)\left[t + \sqrt{\frac{9.81}{(0.0097)^2 \times 9.81}}\left(\ln\left|e^{\left(2\sqrt{0.0097 \cdot 9.81}\right)}\right| - 1\right| - \ln\left|e^{\left(2\sqrt{0.0097 \cdot 9.81}\right)}\right|\right)\right] \quad \text{and find the point of}$$

intersection of the two graphs.

Point of intersection:
x=1000,
y=31.445

```
WINDOW
Xmin=-10
Xmax=50
Xscl=1
Ymin=-10
Ymax=1200
Yscl=0
Xres=1
```

```
Intersection
X=31.44983  Y=1000
```

We thus obtain that at y=1000, x=31.445. The value we found for x is the time it takes the raindrop to reach the ground. This value can now be substituted into the equation for velocity:

$$v = \sqrt{\frac{9.81}{k}}\left(\frac{e^{2\sqrt{kg}v} + 1}{e^{2\sqrt{kg}v} - 1}\right)$$

We thus obtain:

$$v = \sqrt{\frac{9.81}{0.0097}}\left(\frac{e^{2\sqrt{0.0097 \cdot 9.81} \cdot 31.445} + 1}{e^{2\sqrt{0.0097 \cdot 9.81} \cdot 31.445} - 1}\right)$$

$$v = 31.802 \, m/s$$

Again, we find that this is the terminal velocity as

$$\frac{dv}{dt} = g - kv^2$$

equal to zero. Thus:

$$g = kv^2$$
$$v^2 = \frac{g}{k}$$
$$v = \sqrt{\frac{g}{k}}$$

, as explained above. To find the terminal velocity we calculate v when acceleration is

$$v = \sqrt{\frac{9.81}{0.0097}}$$
$$v = 31.802$$

It is thus reasonable to assume that the large raindrops also travel at terminal velocity for most of the time.

The Parachute Problem:

To apply the concept of raindrop modelling to human beings, one could now consider a parachute jump. As I do not know much about skydiving, I first did some research around the basics of it.

A typical parachute jump consists of the individuals jumping out of an aircraft at approximately 4,400 metres altitude. The period of free-fall takes about a minute, then the parachute is opened to slow the landing down, which takes between five and seven minutes.

To model a parachute jump, we can say that the parachutist, just like any other falling object, experiences constant acceleration $g = 9.81 ms^{-2}$, since all objects fall with the same rate of acceleration regardless of their mass.
We also need to account for air resistance and can thus use the same model as before, when modelling the fall of small raindrops.

$$a = \frac{dv}{dt}$$
$$\frac{dv}{dt} = g - kv$$

However, as the size of humans is not comparable to the size of small raindrops, we need to find a different value for k, as the air resistance is not the same in this case. Also, the significance of air resistance will ultimately depend on the position of the skydiver in the air. For example, a skydiver in the spread eagle position will encounter more air resistance than a parachutist, who falls feet or head first, which related back to the cross sectional area of the falling object as explained earlier. As explained earlier, the amount of air resistance is subject to change throughout the free fall, as it is also dependent upon speed. The air resistance is small at the start of the jump and the parachutist accelerates downward. As the speed of the free faller increases, the amount of air resistance goes up as well, thus slowing down acceleration. The air resistance force will eventually be equal and opposite to the downward force due to gravity and at this point the skydiver's speed remains constant. Terminal velocity has thus been reached [6]

In order to develop a model for the parachute problem, we would thus have to divide the free fall into three parts, each with a different amount of air resistance and then also develop a model for the slow descent after the parachute is deployed. The mathematical process however is very simple and the same as in the second model for small raindrops. k is just exchanged for the respective values of the air resistance, that is the constant of proportionality in reference to velocity.

Having extended the mathematical exploration to the problem of a parachute jump, one can now critically reflect upon the work done. We can start by looking at the first model, which merely looks at raindrops as falling objects. It is an unrealistic model as such, as it does not account for air

[6] „KS§ 9K Air resistance and friction", slideshare,
<http://www.slideshare.net/scienceinteractive/ks3-9k-air-resistance-and-friction>, 23 May 2010

resistance. The results could thus be used as a rough idea on how to develop the other models, but other than that, were hypothetical.

The second model, which looks at smaller raindrops, takes air resistance into account and thus gets a lot closer to reality. However, there are certain uncertainties to this model. To begin, we put all raindrops with a diameter smaller that 0.008cm into one group, the results are thus not specific to the respective size of the raindrop. In addition, we made the assumption that the amount of air resistance is always that same. In reality, the amount of air resistance is subject to a number of factors. As stated earlier, it depends upon the cross sectional area and speed of the object and thus changes throughout the fall. However, this is only a minor inaccuracy, as we found out that raindrops reach terminal velocity after a very short period of time and thus travel at terminal velocity, and thus the same speed, for almost the entire fall. The air resistance, however, can also be influenced by environmental factors, such as a strong wind.

These uncertainties are the same for the third model, looking at larger raindrops.

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Example 7: Modelling rainfall

Assessment

Criterion	A	B	C	D	E (SL)	E (HL)	Total (SL)	Total (HL)
Achievement level awarded	3	2	3	2	6	5	16	15
Maximum possible achievement level	4	3	4	3	6	6	20	20

Comments

Criterion A: Communication

A3—Although the communication is good, it is not concise enough for a level 4.

Criterion B: Mathematical presentation

B2—There are many notation errors, but not enough to award only a level 1.

Criterion C: Personal engagement

C3—The student has created some examples.

Criterion D: Reflection

D2—The reflection is meaningful, but not critical.

SL Criterion E: Use of mathematics

E6—Thorough knowledge and understanding have been demonstrated. Accuracy errors are not penalized given the level of understanding demonstrated.

HL Criterion E: Use of mathematics

E5—Thorough knowledge and understanding have been demonstrated, but it lacks the precision required for a level 6.

General comments

Background information from the teacher:

"The student was interested in the stimulus 'weather' and said that she wanted to look into rainfall and to see whether this could be extended to other falling objects.

Once research had started, the student developed differential equations to explain rainfall, but soon found out that she did not have enough knowledge to solve one of the equations. She taught herself how to separate algebraic fractions into partial fractions, which helped her to find the solution she was after. Having supervised the student throughout the process, I can confirm that the student was very engaged with the task and all the work produced is her own."