MM 2 : Related Rates and Application
Name: ________________
Period: _________

(1) Related Rates
A spherical balloon is being filled with a gas in such a way that when the radius is 2 ft, the radius is increasing at the rate of 1/6 ft/min. How fast the volume changing at this time?

(2) Moving Shadow
A person 6 ft tall is walking away from a streetlight 20 ft high at the rate of 7 ft/s. At what rate is the length of the person's shadow increasing?

(3) Leaning Ladder
A bag is tied to the top of a 5-m ladder resting against a verticalwall. Suppose the ladder begins sliding down the wall in such a way that the foot of the ladder is moving away from the wall. How fast is the bag descending at the instant the foot of the ladder is 4 m from the wall and the foot is moving away at the rate of 2m/s?

(4) The Water Level in a Cone-Shaped Tank

A tank filled with water is in the shape of an inverted cone 20 ft high with a circular base (on top) whose radius is 5 ft. Water is running our of the bottom of the tank at the constant rate of 2 cubic ft/min. How fast is the water level falling when the water is 8 ft deep?

(5) Modeling with an angle of Elevation

Every day, a flight to Los Angles flies directly over my home at a constant altitude of 4 mi. If I assume that the plane is flying at a constant speed of 400 mi/h, at what rate is the angle of elevation of my line of sight changing with respect to time when the horizontal distance between the approaching plane and my location is exactly 3 mi?

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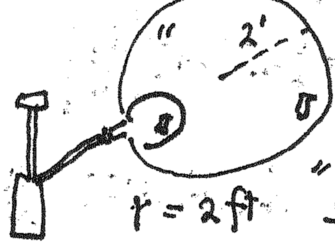
(1) Related Rates

A spherical balloon is being filled with a gas in such a way that when the radius is 2 ft, the radius is increasing at the rate of 1/6 ft/min. How fast the volume changing at this time?



$r = 2\text{ ft}$ $\qquad \frac{dV}{dt} = ?$

$\frac{dr}{dt} = \frac{1}{6}\,\frac{\text{ft}}{\text{min}}$

$V_{sphere}(t) = \frac{4}{3}\pi r(t)^3$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}r^2)\cdot\frac{dr}{dt}$

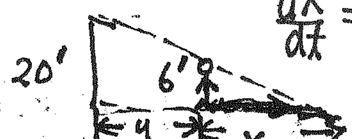
$\frac{dV}{dt} = 4\pi(2)^2\cdot\frac{1}{6}$

$= \boxed{\frac{8}{3}\pi\,\frac{\text{ft}^3}{\text{min}}}$

$(r^3)' = \boxed{\frac{d(r^3)}{dr}\cdot\frac{dr}{dt}}$

(2) Moving Shadow

A person 6 ft tall is walking away from a streetlight 20 ft high at the rate of 7 ft/s. At what rate is the length of the person's shadow increasing?



$\frac{dx}{dt} = ?$

$\frac{dy}{dt} = 7\,\frac{\text{ft}}{\text{s}}$

$\frac{\cancel{6}\,3}{\cancel{20}\,10} = \frac{x}{x+y}$

$3x + 3y = 10x$

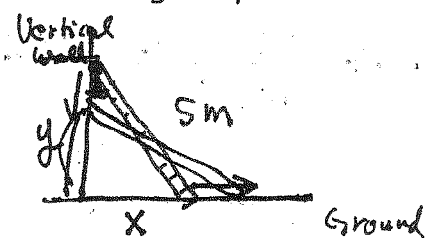
$3y = 7x$

$3\frac{dy}{dt} = 7\frac{dx}{dt} \Rightarrow 3\cdot\cancel{7} = \cancel{7}\cdot\frac{dx}{dt}$

$\frac{dx}{dt} = 3\,\frac{\text{ft}}{\text{sec}}$

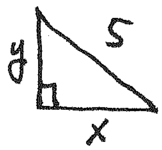
(3) Leaning Ladder

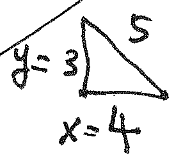
A bag is tied to the top of a 5-m ladder resting against a verticalwall. Suppose the ladder begins sliding down the wall in such a way that the foot of the ladder is moving away from the wall. How fast is the bag descending at the instant the foot of the ladder is 4 m from the wall and the foot is moving away at the rate of 2m/s?



$\frac{dx}{dt} = 2\,\frac{\text{m}}{\text{s}}$ $\qquad \frac{dy}{dt} = ?$

$x = 4\text{ m}$





$x^2 + y^2 = 25$

$2x\cdot\frac{dx}{dt} + 2y\boxed{\frac{dy}{dt}} = 0$

?

$\cancel{2}\cdot4\cdot2 + \cancel{2}\cdot3\cdot\frac{dy}{dt} = 0$

$\boxed{\frac{dy}{dt} = \frac{-8}{3}\,\frac{\text{m}}{\text{s}}}$

(4) The Water Level in a Cone-Shaped Tank

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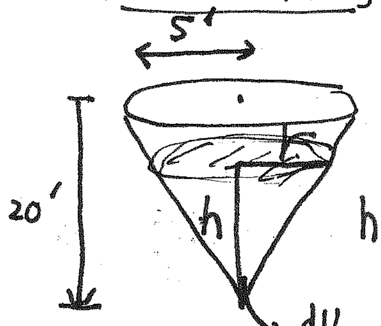


$\frac{dh}{dt} = ?$

$h = 8$ ft

$\frac{dV}{dt} = \frac{2 ft^3}{min}$

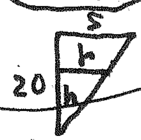
$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 \cdot h$

$\frac{5}{20} = \frac{r}{h}$

$\frac{1}{4} = \frac{r}{h}$

$r = \frac{1}{4}h$

$r = \frac{1}{4}h$



$V = \frac{1}{48}\pi h^3$

$\Rightarrow \frac{dV}{dt} = \frac{1}{48}\pi(3h^2)\boxed{\frac{dh}{dt}}$

Subtitute

$\Rightarrow -2 = \left(\frac{1}{48}\right)(\pi)(3 \cdot 8^2)\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{-1}{2\pi} \frac{ft}{min}$

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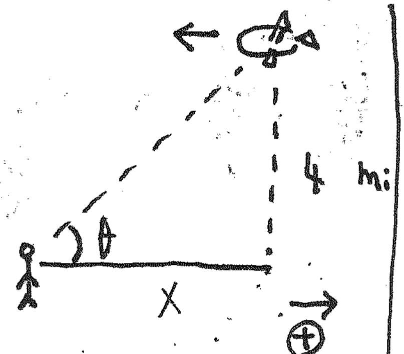
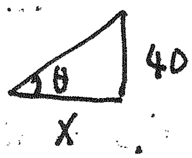


$\frac{dx}{dt} = -400 \frac{mi}{hr}$

$x = 3$ mi

$\frac{d\theta}{dt} = ?$



$\tan\theta = \frac{40}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{40}{x}\right)$

$\frac{d\theta}{dt} = \left(\frac{1}{1+\left(\frac{4}{x}\right)^2}\right)\left(\frac{-4}{x^2}\right) \cdot \frac{dx}{dt}$

Subtitute

$\frac{d\theta}{dt} = \left(\frac{1}{1+\left(\frac{4}{3}\right)^2}\right)\left(\frac{-4}{3^2}\right)(-400)$

$\frac{d\theta}{dt} = 64 \frac{rad}{hr}.$