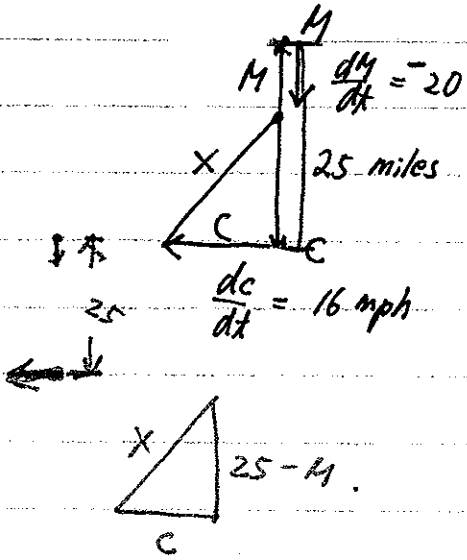
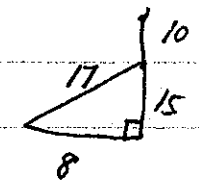
Related Rate Review Answers.

#1.





after $t = .5$ hr.

$M = (20)(.5) = 10$ miles $C = (16)(.5) = 8$ miles $X = 17$ miles

$X^2 = C^2 + (25-M)^2$

$= C^2 + (25)^2 - 50M + M^2$

$\frac{dX}{dt} = ?$

$2X\left[\frac{dX}{dt}\right] = 2C \cdot \frac{dC}{dt} - 50\frac{dM}{dt} + 2M\frac{dM}{dt}$.

$(2)(17)\cdot\frac{dX}{dt} = (2)(8)(16) - (50)(-20) + (2)(10)(-20)$

$\boxed{\frac{dX}{dt} = 25.2 \text{ miles/hr}}$

#2. a. $V_{cylinder} = \pi r^2 h$ ($r = 3$ constant)

$V = \pi \cdot 9 \cdot h$

$\left(\frac{dV}{dt} = 10\frac{ft^3}{min}\right)$ $\frac{dV}{dt} = 9\pi \cdot \frac{dh}{dt}$

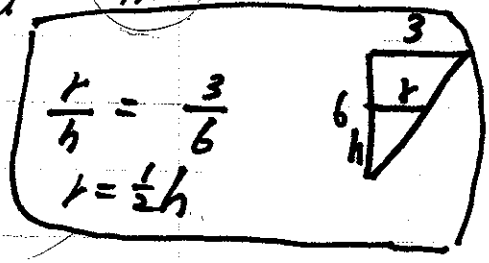
$\frac{dh}{dt} = \frac{10}{9\pi}$ in/min

b. $V = \frac{1}{3}\pi r^2 h$

$\frac{dh}{dt} = ?$

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{\pi}{12}h^3$



$\frac{r}{h} = \frac{3}{6}$

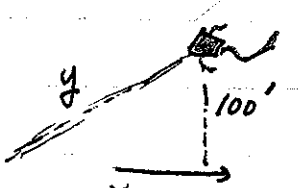
$r = \frac{1}{2}h$

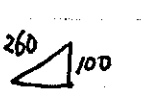
$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$ $\Rightarrow -10 = \frac{\pi}{4}(5)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \left(-10\right)^2\left(\frac{4}{\pi}\right)\left(\frac{1}{25}\right)$

$\frac{dh}{dt} = \frac{-8}{5\pi}$ in/min

#7.





$y^2 = 100^2 + x^2$

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

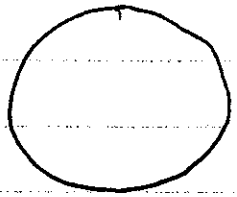
$260 \cdot \frac{dy}{dt} = (240)(15)$

$\frac{dx}{dt} = 15\frac{ft}{sec}$

$y = 260'$

$dy = ?$

$\frac{dy}{dt} = \frac{-180}{13}$ ft/sec $= -13.8$ ft

#3. 

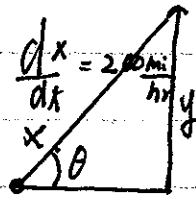
$\frac{dA}{dt} = 6\ mi^2/hr$ $\quad A = \pi r^2$

$A = 9\ mi^2 = \pi r^2$ $\quad \frac{dA}{dt} = 2\pi \cdot r \cdot \frac{dr}{dt}$

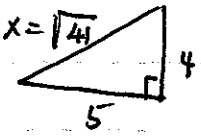
$\frac{dr}{dt} = ?$ $\quad r^2 = \frac{9}{\pi}$ $\quad 6 = (2\pi)\left(\frac{3}{\sqrt{\pi}}\right)\frac{dr}{dt}$

$r = \frac{3}{\sqrt{\pi}}$ $\quad 1 = \sqrt{\pi} \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{\sqrt{\pi}}\ mi/hr$

#4. 

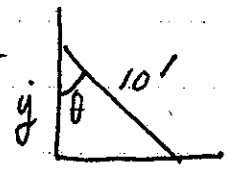
$\frac{dy}{dt} = ?\ \frac{mi}{hr}$ $\quad y = 4$ miles.



$y^2 + 5^2 = x^2$

$2y \cdot \frac{dy}{dt} = 2x \cdot \frac{dx}{dt}$

$2 \cdot \cancel{x} \cdot \frac{dy}{dt} = (2)(\sqrt{41})(200)$ $\quad \frac{dy}{dt} = 320\ \frac{mi}{hr}$.

#5 

$\frac{dx}{dt} = 2\ \frac{ft}{sec}$. $\quad \cos\left(\frac{\pi}{4}\right) = \frac{x}{10}$, $x = \frac{\sqrt{2}}{2} \cdot 10^5 = 5\sqrt{2}$ $\quad$ or $50\sqrt{41}\ \frac{mi}{hr}$

$\theta = \frac{\pi}{4}$. $\quad \frac{d\theta}{dt} = ?$

a. ① $\sin\theta = \frac{x}{10}$ $\quad$ or. $\theta = \sin^{-1}\frac{x}{10}$ $\quad$ ② $\frac{d\theta}{dt} = \frac{1}{\sqrt{1-\left(\frac{x}{10}\right)^2}}\left(\frac{1}{10}\right)\frac{dx}{dt}$

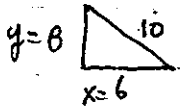
$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{10} \cdot \sec\left(\frac{\pi}{4}\right) \cdot 2 \approx .283\ \frac{rad}{sec}$

$\frac{d\theta}{dt} = \frac{1}{\sqrt{1-\left(\frac{5\sqrt{2}}{10}\right)^2}}\left(\frac{1}{10}\right)(2)$

$= \frac{1}{\sqrt{1-\frac{1}{2}}}\left(\frac{1}{5}\right)$ $\quad \frac{d\theta}{dt} = .283$

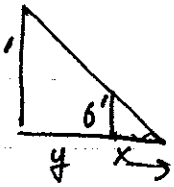
b. $x = 6$ $\quad \frac{dx}{dt} = 2\ \frac{ft}{sec}$ $\quad \frac{dy}{dt} = ?$ $\quad y = 8$



$10^2 = x^2 + y^2$

$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \rightarrow \frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y} = \frac{-6 \cdot 2}{8} = -\frac{3}{2}$

$\Rightarrow \frac{dy}{dt} = \frac{-3}{2}\ \frac{ft}{sec}$

#6. 15' 

$\frac{dy}{dt} = 5\ \frac{ft}{sec}$ $\quad \frac{dx}{dt} = ?$

$\frac{15}{x+y} = \frac{6}{x}$ $\Rightarrow 2x + 2y = 5x$

$2y = 3x$

$\frac{dx}{dt} = \frac{2 \cdot 5}{3} = 10\ ft$ $\quad 2\frac{dy}{dt} = 3\frac{dx}{dt}$

$\frac{dx}{dt} = 10/3$