IB Math 3: Remainder of Taylor Polynomial **Name:** ______________

Warm Up: centered at o

1) **Find the first five terms of the Maclaurin series for** $(1+x)^n$.

$f(x) = (1+x)^n \Rightarrow f(0) = 1$

$f'(x) = n(1+x)^{n-1} \Rightarrow f'(0) = n$

$f''(x) = n(n-1)(1+x)^{n-2} \Rightarrow f''(0) = n(n-1)$

$f'''(0) = n(n-1)(n-2)$

$\Rightarrow M(x) = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \ldots$

2) **Using the series above, find the first five terms of the power series for** $f(x) = \sqrt[3]{1+x}$

$f(x) = (1+x)^{\frac{1}{3}}$ $n = \frac{1}{3}$

$M(x) = 1 + \frac{1}{3}x + \frac{\frac{1}{3}(\frac{1}{3}-1)x^2}{2!} + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)}{3!}x^3 \ldots\ldots$

Alt. Series Remainder : $|S - S_n| \leq a_{n+1}$

$f(x) = T_n(x) + R_n(x)$ where $T_n(x)$ is a Taylor polynomial approximation and $R_n(x)$ is the remainder.

Taylor's Theorem $R_n(x)$

If a function f(x) is differentiable through order n+1 in an interval (I) containing c, then for each x in I, there exists z between x and c such that

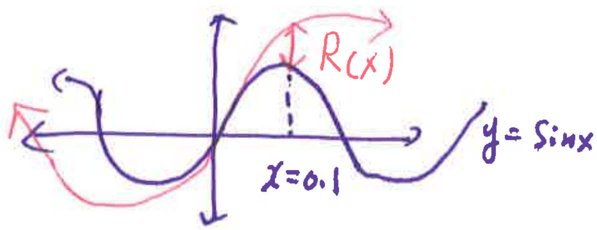
$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} \ldots\ldots \frac{f^n(c)(x-c)^n}{n!} + R_n(x)$$

$$\text{Where } R_n(x) = \frac{f^{n+1}(z)(x-c)^{n+1}}{(n+1)!} \leq \left|f^{n+1}(z_{maz})\right| \bullet \frac{(x-c)^{n+1}}{(n+1)!} \quad \text{and } c \leq z \leq x$$

1. The M_3 for $\sin x$ is $M_3(x) = x - \frac{x^3}{3!}$. Use Taylor's Theorem to approximate $\sin(0.1)$ and determine the accuracy by the Taylor's theorem $R_n(x)$.

as far as x^3

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$



$x = 0.1$

$\sin(0.1) = 0.1 - \frac{(0.1)^3}{3!} + R_3(0.1)$

$R_3(0.1) = \frac{(0.1)^5}{5!} - \frac{(0.1)^7}{7!} \ldots\ldots$

$R_{Max}(0.1) = \frac{(0.1)^5}{5!} \approx 8.3E^{-8}$

$0.0998\bar{3} - 8.3E^{-8} \leq \sin(0.1) \leq 0.0998\bar{3} + 8.3E^{-8}$

$0.0998\bar{3} \leq \sin(0.1) \leq 0.0998\bar{3}$

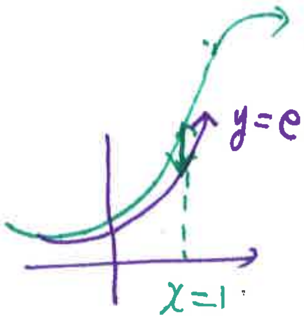
as far as x^4 term

2. Using M_4 for $f(x) = e^x$, estimate e and determine the accuracy by the Taylor's theorem $R_n(x)$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots\cdots$$

$$x = 1 \Rightarrow e = \boxed{1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}} + R_4(1) = 2.708 + R_4(1)$$

$$R_4(1) = \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} \cdots$$



$$R_4(1) = \frac{(e^x)^{5}(c)}{5!}(1-a)^5 \qquad a \leq c \leq 1$$

$$a = 0 \Rightarrow 0 \leq c \leq 1$$

$$= \frac{[e^x(c)](1)^5}{5!}$$

$$R_4(1)_{max} = \frac{(e)}{5!} \quad (c = 1). \qquad e \approx 3.$$

$$= \frac{3}{5!}$$

$$2.708 - \frac{3}{5!} \leq e \leq 2.708 + \frac{3}{5!}$$

$$\boxed{2.688 \leq e \leq 2.808}$$

Exit Slip:

Discuss the concept and help one another within your group. Once your complete this, get a stamp from Mrs. Shim.

1. Write down two major formulas which were introduced related to Taylor's remainder theorem.

2. Demonstrate your understanding of the Taylor's remainder theorem using words and diagram (graphs).