IB Math 3: Reminders of Alternating Series and Taylors Series.

Name: key **Period:** ____

1. Approximate the sum of series $\sum_{k=0}^{\infty} \frac{(-1)^k}{3^k(k!)}$ accurate to 3 decimal places. Justify all process.

$|S - S_n| \le a_{n+1}$ $\quad 0.0005$

$S_n = 1 - \frac{1}{3} + \frac{1}{9 \cdot 2!} - \frac{1}{27 \cdot 3!} + \frac{1}{3^4 \cdot 4!} - \cdots$

$k=0 \quad k=1 \quad k=2$

$a_{n+1} = \frac{1}{3^{n+1}(n+1)!} \le 0.0005 \Rightarrow 3^{n+1}(n+1)! \ge \frac{1}{0.0005}$

$n=4. \quad S_n = 1 - \frac{1}{3} + \frac{1}{18} - \frac{1}{27 \cdot 6} + \frac{1}{3^4 \cdot 4!} \approx .71656$

$R = a_{n+1} = \frac{1}{3^5 \cdot 5!} \approx .000034294.$ $\quad S_4 \approx .717$

$.71656 - .0000343 \le S_4 \le .71656 + .00003429$

2. Find M_4 (~~first 4 none zero terms~~) for $\cos x$. Use Taylor's Theorem to approximate cos(2.1) and determine the accuracy by the Taylor's theorem $R_n(x)$.

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \boxed{.0}$ $\quad x = 2.1$

$n=7$

$\cos(2.1) = 1 - \frac{(2.1)^2}{2!} + \frac{(2.1)^4}{4!} - \frac{(2.1)^6}{6!} + R_6(2.1)$

$= A$

$R_6(2.1) \Rightarrow$ Max. value possible

$\Rightarrow n = 8. \Rightarrow R_6(2.1) = \frac{(\cos x)^{(8)}(z)(2.1)^8}{8!}$ $\quad 0 \le z \le 2.1$

$\cos(z)$

$R_6(2.1) = \frac{(1)(2.1)^8}{8!}$ $\quad \cos(0) = 1$

4. Find a power series for $f(x) = \frac{3x-1}{x^2-1}$ centered at x=0. And find the radius of convergence.

G Series$_\infty$ Sum

$|r| < 1$

$\frac{a_1}{1-r}$

$\cos(2.1) \approx A$

$A - \frac{(2.1)^8}{8!} \le \cos(2.1) \le A + \frac{(2.1)^8}{8!}$

$f(x) = \frac{1-3x}{1-x^2}$

$$f(x) = \frac{A}{x+1} + \frac{B}{x-1} = \frac{3x-1}{x^2-1}.$$

$A = 2$

$B = 1.$

$A(x-1) + B(x+1) = 3x-1$

$x=1$

$B = \frac{2}{2}$ $\quad B=1$

$x=-1$ $\quad A=2$

$$f(x) = \frac{2}{1+x} + \frac{1}{x-1} = \frac{2}{1+x} + \frac{-1}{1-x}.$$

$a_1 = 2$ $\quad a_2 = -1$

$r_1 = -x$ $\quad r_2 = x.$

$$\Rightarrow \sum_{n=0}^{\infty} (2)(-x)^n + \sum_{n=0}^{\infty} (-1)(x)^n$$

$|x| < 1.$

$$= \boxed{\sum_{n=0}^{\infty} (2)(-1)^n(x)^n + \sum_{n=0}^{\infty} (-1)(x)^n}$$

$$= \boxed{\sum_{n=0}^{\infty} \left[2(-1)^n - 1\right] \cdot x^n}$$