Key.

Welcome Back! I hope you had a restful break. I want to thank you for the wonderful 2018. It was my privilege teaching you and I look forward to 2019 as wonderful as 2018! Each of you all is a precious student to me. We were on the vector unit. Today you are just reviewing the previous material and will learn 'cross product' tomorrow. Work on the following questions to refresh what we did on the vector unit.

Name: ______________ Period: ________

1. If $a = \begin{pmatrix} 3 \\ -4 \\ \sqrt{11} \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ 1 \\ \sqrt{11} \end{pmatrix}$.

a. Find vector **c** in the direction of vector a with a magnitude of 5 units.

$|a| = \sqrt{9+16+11} = \sqrt{36} = 6$

$c = 5\left(\frac{3}{6}i - \frac{4}{6}j + \frac{\sqrt{11}}{6}k\right) = \boxed{\frac{5}{2}i - \frac{10}{3}j + \frac{5\sqrt{11}}{6}k}$

b. Find $a \cdot b$.

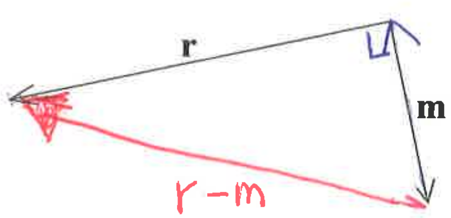
$a \cdot b = (-6 - 4 + 11) = \boxed{1}$

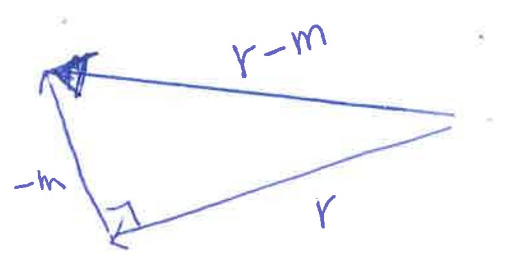
c. Find the angle between a and b. Leave your answer rounded to 3 sig figs in radians.

$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{1}{6 \cdot 4} \Rightarrow \theta = \arccos\left(\frac{1}{6 \cdot 4}\right) = \boxed{1.53 \text{ rad}}$

$|b| = \sqrt{4+1+11} = \sqrt{16} = 4$

3. The following diagram shows two perpendicular vectors **r** and **m**.





a. Let **b** = **r** – **m**. Represent **b** on the diagram above.

b. Given that $\mathbf{r} = 5\mathbf{i} - 6\mathbf{j} + w\mathbf{k}$ and $\mathbf{m} = -4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, find the value of w.

$r \cdot m = 0 \Rightarrow (-20 - 12 + w) = 0 \Rightarrow \boxed{w = +32}$

4. Find k if $\begin{pmatrix} -\frac{1}{3} \\ k \\ \frac{3}{5} \end{pmatrix}$ is a unit vector.

$$\left|-\tfrac{1}{3}i + kj + \tfrac{3}{5}k\right| = \sqrt{\tfrac{1}{9} + k^2 + \tfrac{9}{25}} = 1 \Rightarrow$$

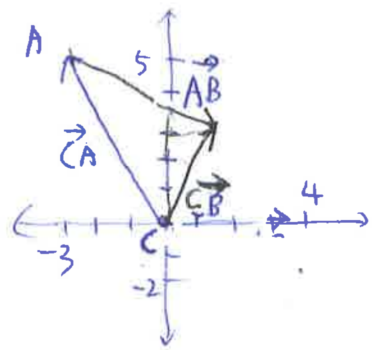
$$\Rightarrow \sqrt{\tfrac{106}{225} + k^2} = 1 \Rightarrow k^2 = \tfrac{119}{225}$$

$$k = \pm\sqrt{\tfrac{119}{225}} = \pm\tfrac{\sqrt{119}}{15}$$

5. Given $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\overrightarrow{OA} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$, find $\overrightarrow{CB}$.

$$\vec{CB} = \vec{CA} + \vec{AB} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 1 \\ 3 \end{pmatrix}}$$



6. Consider points $P(5,-4)$, $Q(8,3)$, and $R(m,-8)$. Find m if $Q\hat{P}R$ is a right angle.

$$\vec{PQ} = \begin{pmatrix} 8 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \quad \Rightarrow Q\hat{P}R = \frac{\vec{PQ} \cdot \vec{PR}}{} = 0$$

$$\vec{PR} = \begin{pmatrix} m \\ -8 \end{pmatrix} - \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} m-5 \\ -4 \end{pmatrix} \quad \Rightarrow 3(m-5) + (7)(-4) = 0$$

$$3m - 15 - 28 = 0 \Rightarrow 3m = \frac{43}{3}$$

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7. A(4, -2, 5), B(13, 1, -2) and C(1, r, t) and collinear. Find r and t.

$$\vec{AB} = t\,(\vec{BC})$$

$$\begin{pmatrix} 13-4 \\ 1-(-2) \\ -2-5 \end{pmatrix} = k \begin{pmatrix} 1-13 \\ r-1 \\ t-(-2) \end{pmatrix} \Rightarrow$$

$$9 = k(-12) \Rightarrow k = \frac{9}{-12} = \frac{3}{-4}$$

$$3 = k(r-1) \Rightarrow 3 = \left(\frac{3}{-4}\right)(r-1) \Rightarrow r-1 = 3\left(\frac{-4}{3}\right)$$

$$r = -4 + 1 = -3$$

$$-7 = k(t+2) \Rightarrow -7 = \left(\frac{3}{-4}\right)(t+2) \Rightarrow t = \frac{22}{3}$$