Notes

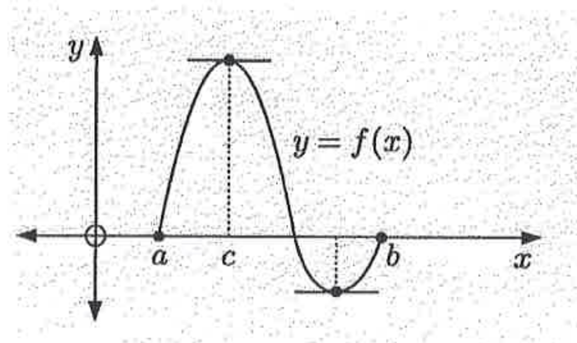
IB Math HL1 Name: _______________ Period: ____

Rolle's Theorem and the Mean Value Theorem

Rolle's Theorem:

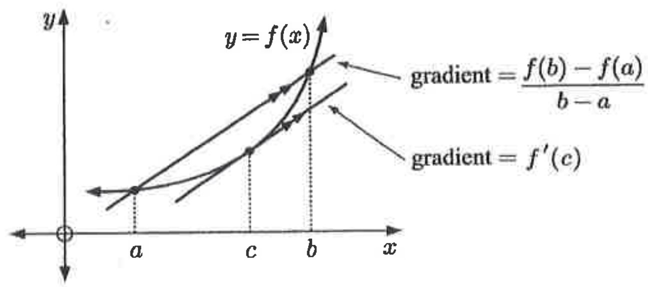
- **Suppose A function $f: D \to R$ is continuous on the closed interval $[a,b]$, and differentiable at the open interval $]a,b[$.**
- **If $f(a) = f(b) = 0$, then there exists a value $c \in]a,b[$ such that** $f'(c) = 0$

OR $f(a) = f(b)$



Mean Value Theorem (MVT):

- **Suppose A function $f: D \to R$ is continuous on the closed interval $[a,b]$, and differentiable at the open interval $]a,b[$, then** $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ **for some number** $c \in]a,b[$.



Example 1) Show that function $f(x) = x^3 + x$ satisfies the hypothesis of the MVT on the closed interval [1, 2], and find the number c between [1, 2] that satisfies the MVT.

MVT: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

$a = 1, \quad b = 2$.

$f'(x) = 3x^2 + 1$

$f(1) = 2 \qquad f(2) = 8 + 2 = 10$

$\Rightarrow f'(c) = 3c^2 + 1 = \dfrac{10 - 2}{2 - 1} = 8$

$3c^2 + 1 = 8. \Rightarrow \boxed{c = \pm\sqrt{\frac{7}{3}}}$

$-\sqrt{\frac{7}{3}}$ is Not in $[1, 2]$

$\therefore \boxed{c = \sqrt{\frac{7}{3}}}$

Example 2) Gvien $f(x) = x^2 - 3x + 2$; Using the Rolle's theorem show that $f'(x) = 0$ exit between (1, 2).

Rolle's theorem:
If $f(1) = f(2)$,
then $f'(c) = 0$ exits on $(1, 2)$

$\Rightarrow f(1) = 1 - 3 + 2 = 0$

$f(2) = 4 - 6 + 2 = 0$

$\Rightarrow f(1) = f(2)$

$\therefore f'(c) = 0$ Must exit on $(1, 2)$

check:
$f'(x) = 2x - 3 = 0$
$x = \frac{3}{2}$
$1 < \frac{3}{2} < 2$

Example 3) Given $f(x) = x^3 - 2x^2 - x + 2$:

a) Find all x-intercepts.

$0 = x^3 - 2x^2 - x + 2$ $\Rightarrow (x-2)(x^2-1) = 0$

$= x^2(x-2) - (x-2) = 0$ $\quad (x-2)(x-1)(x+1) = 0$

$\boxed{x = 2,\ x = 1,\ x = -1}$

b) Find all values of c between the x-intercepts for f(x) that satisfy the conclusion of Rolle's Theorem. Determine if $f(c)$ is the local max or the local min. Justify it by Rolle's Theorem.

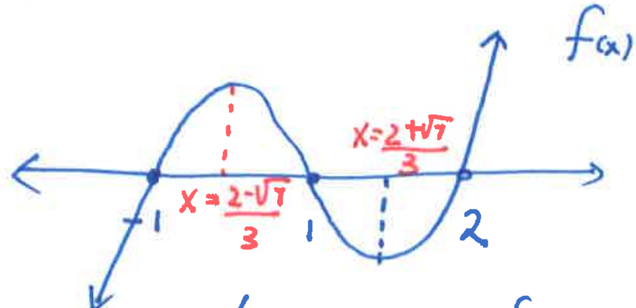
$\frac{df}{dx} = 3x^2 - 4x - 1$

$\left.\frac{df}{dx}\right|_{x=c} = 0 = 3c^2 - 4c - 1$

$c = \frac{4 \pm \sqrt{16 + 4\cdot 3}}{6}$

$= \frac{4 \pm 2\sqrt{7}}{6} = \frac{2 \pm \sqrt{7}}{3}$

$f(x) = x^3 - 2x^2 - x + 2$



We know $f'\left(\frac{2+\sqrt{7}}{3}\right) = 0 \Rightarrow f\left(\frac{2+\sqrt{7}}{3}\right) < 0$

$f'\left(\frac{2-\sqrt{7}}{3}\right) = 0 \Rightarrow f\left(\frac{2-\sqrt{7}}{3}\right) > 0$

$\Rightarrow \therefore$ $\boxed{f\left(\frac{2+\sqrt{7}}{3}\right) \text{ is min},\ f\left(\frac{2-\sqrt{7}}{3}\right) \text{ is Max.}}$

Homework: Do only Circled ones

1 Determine whether or not Rolle's theorem applies to the function f on the given interval $[a, b]$. If Rolle's theorem does apply, find all values $c \in\,]a, b[$ for which $f'(c) = 0$.

- **a** $f(x) = 3x^3 + 5x^2 - 43x + 35, \quad [a, b] = [-5, 2\frac{1}{3}]$
- **b** $f(x) = |x| - 5, \quad [a, b] = [-5, 5]$
- **c** $f(x) = 2 - \frac{1}{x+1}, \quad [a, b] = [-\frac{1}{2}, 7]$
- **d** $f(x) = \begin{cases} -2x - 5, & x < -1 \\ x^2 - 4, & x \geqslant -1 \end{cases}, \quad [a, b] = [-2\frac{1}{2}, 2]$

2 For each of the following functions, find:

- **i** the number of real zeros of the derivative $f'(x)$, as guaranteed by Rolle's theorem
- **ii** the exact number of real zeros of the derivative $f'(x)$.

- **a** $f(x) = (x-1)(x-2)(x-4)(x-5)$
- **b** $f(x) = (x-1)^2(x^2-9)(x-2)$
- **c** $f(x) = (x-1)^2(x^2+9)(x-2)$

3 Show that the given function f satisfies the MVT on the given interval $[a, b]$. Find all values of c such that $f(b) - f(a) = f'(c) \times (b - a)$.

- **a** $f(x) = x^3, \quad [a, b] = [-2, 2]$
- **b** $f(x) = \sqrt{x-2}, \quad [a, b] = [3, 6]$
- **c** $f(x) = x + \frac{1}{x}, \quad [a, b] = [1, 3]$