2x2 Row Reduction

Row Reduction: A method for solving systems of linear equations using augmented matrices.

Process

Given a system of equations	Set up an augmented matrix.	Use row operations to get zero(s) in the lower left corner (echelon form).	Work from the bottom up to find the solution for each variable.
$2x-3y=5$ $x-6y=4$	$\begin{bmatrix} 2 & -3 & 5 \\ 1 & -6 & 4 \end{bmatrix}$	$\begin{bmatrix} \_ & \_ & \_ \\ 0 & \_ & \_ \end{bmatrix}$	Change the bottom row back to an equation. Solve for y. Now use the top row/equation to solve for x.

$\Rightarrow \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \end{bmatrix}$.

Row Operations

A row can be... swapped; replaced by a non-zero multiple of itself; replaced by itself (or a multiple) plus r a multiple of another row.

Use Row Reduction to solve each system.

1. $\begin{matrix} R_1 \\ R_2 \end{matrix}\begin{bmatrix} 2x-3y=5 \\ x-6y=4 \end{bmatrix} \Rightarrow \begin{matrix} R_1 \\ R_2 \end{matrix}\begin{bmatrix} 2 & -3 & 5 \\ 1 & -6 & 4 \end{bmatrix} \xrightarrow{R_1-2R_2 \to R_2} \begin{bmatrix} 2 & -3 & 5 \\ 0 & 9 & -3 \end{bmatrix} \xrightarrow{\frac{R_2}{9}\to R_2} \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -\frac{1}{3} \end{bmatrix} \xrightarrow{R_1+3R_2 \to R_1} \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & -\frac{1}{3} \end{bmatrix}$

$\xrightarrow{\frac{R_1}{2}\to R_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -\frac{1}{3} \end{bmatrix}$

$x=2,\ y=-\frac{1}{3}$.

3x3 Row Reduction

Row Reduction: A method for solving systems of linear equations using augmented matrices.

Process

Given a system of equations	Set up an augmented matrix.	Use row operations to get zero(s) in the lower left corner (echelon form).	Work from the bottom up to find the solution for each variable.
$x+3y-z=15$ $2x+y+z=7$ $x-y-2z=0$	$\begin{bmatrix} 1 & 3 & -1 & 15 \\ 2 & 1 & 1 & 7 \\ 1 & -1 & -2 & 0 \end{bmatrix}$	② ④ ③ $\begin{bmatrix} \_ & \_ & \_ & \_ \\ 0 & \_ & \_ & \_ \\ 0 & 0 & \_ & \_ \end{bmatrix}$	$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ve for z first.

Row Operations

A row can be... swapped; replaced by a non-zero multiple of itself; replaced by itself (or a multiple) plus a multiple of another row.

$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}\begin{bmatrix} 1 & 3 & -1 & 15 \\ 2 & 1 & 1 & 7 \\ 1 & -1 & -2 & 0 \end{bmatrix} \Rightarrow \xrightarrow{R_1-R_3 \to R_3} \begin{bmatrix} 1 & 3 & -1 & 15 \\ 2 & 1 & 1 & 7 \\ 0 & 4 & 1 & 15 \end{bmatrix} \xrightarrow{2R_1-R_2 \to R_2} \begin{bmatrix} 1 & 3 & -1 & 15 \\ 0 & 5 & -3 & 23 \\ 0 & 4 & 1 & 15 \end{bmatrix} \Rightarrow \xrightarrow{4R_2-5R_3 \to R_3} \begin{bmatrix} 1 & 3 & -1 & 15 \\ 0 & 5 & -3 & 23 \\ 0 & 0 & -17 & 17 \end{bmatrix}$

$\xrightarrow{\frac{R_3}{-17}\to R_3}$

$\Rightarrow \xrightarrow{\substack{3R_3+R_2 \to R_2 \\ \frac{R_2}{5}\to R_2}} \begin{bmatrix} 1 & 3 & -1 & 15 \\ 0 & 5 & 0 & 20 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow \xrightarrow{R_1+R_3 \to R_1} \begin{bmatrix} 1 & 3 & 0 & 14 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow \xrightarrow{R_1-3R_2 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$x=2$
$y=4$
$z=-1$

Use Row Reduction to solve each system.

1. $\begin{matrix} x+3y-z=15 \\ 2x+y+z=7 \\ x-y-2z=0 \end{matrix}$ $(2,4,-1)$

2. $\begin{matrix} 5x+y+2z=48 \\ 3x-2y-z=40 \\ -2x+4y-3z=-50 \end{matrix}$ $(10,-6,2)$

3. $\begin{matrix} 5x+y-4z=-4 \\ -3y-6z=-21 \\ -x-y-z=-6 \end{matrix}$ $(5,-5,6)$

4. $\begin{matrix} 4x-4y+2z=-14 \\ 4x+2y=14 \\ -3y+z=-10 \end{matrix}$ No solution.

5. $\begin{matrix} 3x+6y=-4 \\ x+y-z=-2 \\ 9x-12y+15z=28 \end{matrix}$ $\left(0, -\frac{2}{3}, \frac{4}{3}\right)$

6. $\begin{matrix} 2x+y-2z=6 \\ -6x-2y+z=-6 \\ 8x+3y-3z=12 \end{matrix}$ Infinitely many solutions.