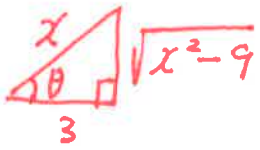
IB Math HL2: Differential Equations Name: key

1. Separable Equations:

$\boxed{M(x)dx + N(y)dy = 0}$

Example 1) $9dx - x\sqrt{x^2-9}dy = 0$



$9dx = x\sqrt{x^2-9}\,dy$ $3\sec\theta = x \Rightarrow dx = 3\sec\theta\tan\theta\,d\theta$.

$3\tan\theta = \sqrt{x^2-9}$.

$\int \frac{dx}{x\sqrt{x^2-9}} = \int \frac{1}{9}dy$.

$\Rightarrow \int \frac{3\sec\theta\tan\theta\,d\theta}{3\sec\theta\cdot 3\tan\theta} = \int \frac{1}{9}dy$.

$\Rightarrow \int 3\frac{d\theta}{3} = \int dy$.

$3\theta = y + C$.

$\boxed{y = 3\operatorname{arcsec}\frac{x}{3} + C}$

Example 2) $xy\frac{dy}{dx} = x^2 + y^2 + x^2y^2 + 1$

$xy\cdot\frac{dy}{dx} = x^2(1+y^2) + (1+y^2)$

$xy\cdot\frac{dy}{dx} = (1+y^2)(1+x^2)$

$\Rightarrow \int \frac{y\,dy}{1+y^2} = \int \frac{(1+x^2)}{x}\cdot dx$.

$u = 1+y^2$

$du = 2\cdot y\,dy$.

$\frac{1}{2}du = y\,dy$.

$\frac{1}{2}\ln(1+y^2) = 2\ln x + \frac{1}{2}x^2 + 2C$.

$\ln(1+y^2) = \ln x^2 + x^2 + 2C$.

$1+y^2 = e^{[\ln x^2 + x^2 + 2C]}$

$1+y^2 = A\cdot e^{(\ln x^2 + x^2)}$

$y^2 = A\cdot e^{(\ln x^2 + x^2)} - 1$

$\boxed{y = \pm\sqrt{A\,e^{(\ln x^2 + x^2)} - 1}}$

OR

$\boxed{y = \pm\sqrt{Ax^2 e^{x^2} - 1}}$